

BUKTI KORESPONDENSI PADA JURNAL NASIONAL
TERAKREDITASI KEMENRISTEKDIKTI SINTA 2

Yth. Penilai Usulan PAK

Bersama ini kami sampaikan proses submit sampai publish artikel ilmiah saya yang berjudul

No	Karya Ilmiah	Judul	Identitas Karya Ilmiah
1	Jurnal Nasional Terakreditasi Sinta 2	The Odd Harmonious Labeling of Layered Graphs	JTAM Jurnal Teori dan Aplikasi Matematika April 2023, Vol. 7 No. 2, Hal 339-348 2597-7512 (Print), 2614-1175 (Online) Universitas Muhammadiyah Mataram https://journal.ummat.ac.id/index.php/jtam/article/view/12506/pdf

Kami sampaikan bukti korespondensi yang sejujurnya untuk dapat dipergunakan sebagai bahan pertimbangan syarat khusus untuk kenaikan jabatan akademik Lektor 200 menjadi Lektor 300

Klaten, 20 Agustus 2024

Hormat Saya,

Fery Firmansah, S.Si., M.Si.

NIP/ NIDN. 198808302015041004/0030088802

Submit Artikel ke Jurnal JTAM (19 Desember 2022)

The screenshot shows the submission page for article #12506. The header includes the journal logo and ISSN information: ISSN 2597-7512 (Print) and ISSN 2614-1175 (Online). The submission details are as follows:

Authors	Fery Firmansah
Title	The Odd Harmonious Labeling of Layered Graphs
Original file	12506-41251-1-5M.DOCX 2022-12-19
Supp. files	None
Submitter	Fery Firmansah
Date submitted	December 19, 2022 - 06:17 AM
Section	Articles
Editor	Mahsup Mahsup
Abstract Views	47

The status is "Published" in Vol 7, No 2 (2023): April, initiated on 2023-04-08. A sidebar on the right contains a "QUICK MENU" with links to Journal History, Editorial Team, Reviewer Team, Focus and Scope, Author Guidelines, Publication Ethics, Open Access Policy, Peer Review Process, and Screening Plagiarism.

This screenshot shows the "Submission Metadata" section of the article page. The author information is:

Authors	Fery Firmansah
Name	Fery Firmansah
Affiliation	Department of Mathematics Education, Widya Dharma University, Klaten
Country	Indonesia
Bio Statement	—
Principal contact for editorial correspondence.	

The abstract text is: "The Odd Harmonious Labeling of Layered Graphs. Graphs that have the properties of odd harmonious labeling are odd harmonious graphs. The research objective of this paper is to obtain odd harmonious labeling on layered graph $C(x,y)$ and layered graph $D(x,y)$. The research used in this paper is a qualitative method. The research flow consists of data collection, processing, and analysis. The data collection stage consists of constructing the definition of the new class graph, the data processing stage consists of constructing the vertex labeling and edge labeling, and the data analysis stage consists of constructing the theorem and proving it. The research results show that the layered graph $C(x,y)$ and layered graph $D(x,y)$ fulfill odd harmonious labeling. Such that the layered graph $C(x,y)$ and layered graph $D(x,y)$ are odd harmonious graphs. The benefit of this research is to add new properties of odd harmonious graphs. In addition, it does not rule out the possibility that this research can be developed again both in theory and application."

Indexing Agencies: Harmonious labeling; Layered graphs; Odd harmonious graphs; Odd harmonious labelling. en

References: Mathematics Education Study Program, Faculty of Teacher Training and Education, Widya Dharma University Klaten, Indonesia

References: Abdel-Aal, M. E. (2013). Odd Harmonious Labelings of Cyclic Snakes. International Journal on Applications of Graph Theory In Wireless Ad Hoc Networks And Sensor Networks, 5(3), 1-11. <https://doi.org/10.5121/ijgraphoc.2013.5301>
Abdel-Aal, & Seoud. (2016). Further Results on Odd Harmonious Graphs. International Journal on Applications of Graph Theory In Wireless Ad Hoc Networks And Sensor Networks, 8(3/4), 1-14. <https://doi.org/10.5121/ijgraphoc.2016.8401>

Scopus Citation Analysis sidebar includes: USER (feryfirmansah), DOWNLOAD DOCUMENT (JTAM Template, Registration Form), and GOOGLE SCHOLAR JTAM.

Review Artikel oleh Reviewer A (15 Maret 2023) dan Reviewer B (20 Februari 2023)

The screenshot shows the submission review page for article #12506 on the JTAM website. The page header includes the journal logo, ISSN numbers (2597-7512 Print, 2614-1175 Online), and the publisher's name, Universitas Muhammadiyah Mataram. A navigation menu is located below the header. The main content area is titled "#12506 Review" and includes tabs for "SUMMARY", "REVIEW", and "EDITING". Under the "Submission" section, the author is listed as Fery Firmansah, and the title is "The Odd Harmonious Labeling of Layered Graphs". The "Peer Review" section shows a "Round 1" review with a version number 12506-41252-2-RV.DOCX, dated 2022-12-22. It lists the review initiation date (2022-12-24), the last modification date (2023-03-15), and the reviewer information: Reviewer A (12506-46257-1-RV.DOCX, 2023-03-15) and Reviewer C (12506-45254-1-RV.DOCX, 2023-02-20). A "Send message" button is visible on the left. A sidebar on the right contains a "QUICK MENU" with links to Journal History, Editorial Team, Reviewer Team, Focus and Scope, Author Guidelines, Publication Ethics, Open Access Policy, Peer Review Process, and Screening Plagiarism. The browser's taskbar at the bottom shows several open files related to the article and the system clock at 10:12 on 04/05/2023.

The screenshot shows an email from the JTAM Editorial Team. The email subject is "[JTAM] Editor Decision" and it is addressed to the user. The email content states: "We have reached a decision regarding your submission to JTAM (Jurnal Teori dan Aplikasi Matematika), 'The Odd Harmonious Labeling of Layered Graphs'. Our decision is: Revisions Required. Note: Download file Reviewer A & B, then revise it. Please upload the revision before March 23, 2023." The email footer provides contact information for the JTAM Editorial Team, including the email address jtam.ummat@gmail.com and the Editor in Chief, Svahanudin I+62 878640038471. The browser's taskbar at the bottom shows several open files related to the article and the system clock at 10:14 on 04/05/2023.

Catatan Reviewer A

12506-46257-1-RV (1) [Compatibility Mode] - Word

File Home Insert Design Layout References Mailings Review View Help Nitro Pro Tell me what you want to do

Spelling & Thesaurus Grammar Proofing Word Count ABC 123 Check Accessibility Language Translate Language New Comment Delete Previous Next Show Comments Track Changes Reviewing Pane Accept Reject Previous Next Compare Block Authors Restrict Editing Hide Ink

JTAM (Jurnal Teori dan Aplikasi Matematika)
<http://journal.ummat.ac.id/index.php/itam>
 p-ISSN 2597-7512 | e-ISSN 2614-1175
 Vol. X, No. Y, Month, 20XX, pp. XX-YY

The Odd Harmonious Labeling of Layered Graphs

ABSTRACT

Article History:
 Received : D-M-20XX
 Revised : D-M-20XX
 Accepted : D-M-20XX
 Online : D-M-20XX

Keyword :
 Harmonious labeling;
 Layered graphs;
 Odd harmonious graphs;
 Odd harmonious labeling;

The research objective of this paper is to obtain odd harmonious labeling on layered graph $C(x,y)$ and layered graph $D(x,y)$. The research method used in this paper is a qualitative research method. The research flow consists of data collection, processing, and analysis. The research results in this paper show that the layered graph $C(x,y)$ and layered graph $D(x,y)$ fulfill odd harmonious labeling. Such that the layered graph $C(x,y)$ and layered graph $D(x,y)$ are odd harmonious graphs.

Comments

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M Please state why this research is important?

Page 1 of 9 2113 words Indonesian Accessibility: Unavailable 13:20 20/08/2024

12506-46257-1-RV (1) [Compatibility Mode] - Word

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Spelling & Thesaurus Grammar Proofing Word Count ABC 123 Check Accessibility Language Translate Language New Comment Delete Previous Next Show Comments Track Changes Reviewing Pane Accept Reject Previous Next Compare Block Authors Restrict Editing Hide Ink

Figure 1. Flowchart research methodology

C. RESULT AND DISCUSSION

1. Construction of layered graph $C(x,y)$ and its odd harmonious labeling

The following definition is given for a layered graph $C(x,y)$

Definition 1.
 Layered graph $C(x,y)$ with $x \geq 1$ and $y \geq 1$ is a graph with $V(C(x,y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y+1\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y\}$ and $E(C(x,y)) = \{a_i^j b_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j-1} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_{i-1}^{j+1} a_i^j | 2 \leq i \leq x\}$.
 In such a way that it is obtained $p = |V(C(x,y))| = 3xy + x$ and $q = |E(C(x,y))| = 4xy + x - 1$. The following is given the construction of the layered graph $C(x,y)$.

Comments

M March 15, 2023
 resenation of Results and Discussion please follow Figure 1 that you have compiled.
 1. Selection data, explain the results
 2. Construction of Graph, explain the result
 and so on. You must consistently follow the method that you have laid out.

M You're writing an article, not a brief note. So please describe each result at each stage. For each picture, please provide a description of its purpose. Don't just write down theorems and proofs.

Page 1 of 9 2113 words Indonesian Accessibility: Unavailable 13:22 20/08/2024

Catatan Reviewer B

The screenshot shows the Microsoft Word interface with the document "12506-45254-1-RV (1) [Compatibility Mode] - Word". The document is in Indonesian. The title is "The Odd Harmonious Labeling of Layered Graphs". The journal information is: JTAM (Jurnal Teori dan Aplikasi Matematika), <http://journal.ummat.ac.id/index.php/jtam>, p-ISSN 2597-7512 | e-ISSN 2614-1175, Vol. X, No. Y, Month 20XX, pp. XX-YY.

ABSTRACT

The research objective of this paper is to obtain odd harmonious labeling on layered graph $C(x,y)$ and layered graph $D(x,y)$. The research method used in this paper is a qualitative research method. The research flow consists of data collection, processing, and analysis. The research results in this paper show that the layered graph $C(x,y)$ and layered graph $D(x,y)$ fulfill odd harmonious labeling. Such that the layered graph $C(x,y)$ and layered graph $D(x,y)$ are odd harmonious graphs.

Article History:
Received : D-M-20XX
Revised : D-M-20XX
Accepted : D-M-20XX
Online : D-M-20XX

Keyword:
Harmonious labeling
Layered graphs:
Odd harmonious graphs:
Odd harmonious labeling:

Comments

USWA February 20, 2023
[This abstract is too short, generally 150-250 words. Please add more so that the overall content of the paper can be explained here. Because your paper needs to be well understood by the reader. You can add a sentence about Introduction, a detailed explanation of the methods you used, an explanation of the results you got, and the conclusions and benefits of your research results.]

The screenshot shows the Microsoft Word interface with the document "12506-45254-1-RV (1) [Compatibility Mode] - Word". The document is in Indonesian. The title is "The Odd Harmonious Labeling of Layered Graphs".

D. CONCLUSION AND SUGGESTIONS

Based on the results and discussion, a new graph class definition construction is obtained for the layered graphs $C(x,y)$ and layered graphs $D(x,y)$. Furthermore, it has been proven that layered graph $C(x,y)$ and layered graph $D(x,y)$ fulfill odd harmonious labeling so that they are odd harmonious graphs.

Suggestions for future research, this research can be continued by finding new graph classes that satisfy the properties of odd harmonious labeling.

ACKNOWLEDGEMENT

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Abdel-Aal, & Seoud. (2016). Further Results on Odd Harmonious Graphs. *International Journal on Applications of Graph Theory In Wireless Ad Hoc Networks And Sensor Networks*, 8(3/4), page?. <https://doi.org/10.5121/jgraphoc.2016.8401>

Febriana, F., & Sugeng, K. A. (2020). Odd harmonious labeling on squid graph and double squid graph. *Journal of Physics: Conference Series*, 1538(1), page?. <https://doi.org/10.1088/1742-6596/1538/1/012015>

Firmansah, F. (2017). The Odd Harmonious Labeling on Variation of the Double Quadrilateral Windmill Graphs. *Jurnal Ilmu Dasar*, 18(2), 109. <https://doi.org/10.19184/jid.v18i2.5648>

Firmansah, F. (2020a). Pelabelan Harmonis Ganjil pada Graf Bunga Double Quadrilateral. *Jurnal Ilmiah Sains*, 20(1), page?. <https://doi.org/10.35799/jis.20.1.2020.27278>

Firmansah, F. (2020b). Pelabelan Harmonis Ganjil pada Graf Ular Jaring Berlipat. *Sainmatika*:

Comments

USWA February 20, 2023
[Your conclusion is too similar to Abstract. Please re-construct it. Please make the conclusion refer to the purpose of your research.]

Submit Hasil Revisi oleh Penulis (17 Maret 2023)

The screenshot shows the 'Peer Review' section of the journal submission system. It details the 'Round 1' review process, including the submission date (2022-12-22) and the reviewer's feedback (2023-03-15). The 'Editor Decision' section shows the submission was accepted on 2023-03-23. A sidebar on the right lists various journal policies and a user profile for 'feryfirmansah'. At the bottom, there is a grid of logos for various indexing services like DOAJ, SINTA, EBSCO, Google, and others.

Artikel Diterima (23 Maret 2023)

The screenshot displays an email from 'JTAM Official' with the subject 'Accepted & Published Fees [JTAM]'. The email is dated March 23, 2023, and is addressed to the author. The main body of the email states that the author's paper (ID 12506) has been accepted for publication in JTAM Vol. 7, No. 2, April 2023. It specifies a publication fee of Rp1,500,506 and provides the contact information for the Editor-in-Chief, Syaharuddin. The email also includes links to the journal's website and its SINTA 2 accreditation.

Pembayaran Artikel

The screenshot shows a Gmail interface with a search bar containing 'jtam'. The email list on the left includes 'Kotak Masuk' (229), 'Berbintang', 'Ditunda', 'Terkirim', 'Draf' (8), and 'Selengkapnya'. The main email is from 'Fery Firmansah <firmansahmath@gmail.com>' dated 'Kam, 23 Mar, 12.39'. The subject is 'Pembayaran JTAM an Fery Firmansah ID 12506'. The body text reads: 'Berikut kami lampirkan bukti pembayaran Jurnal JTAM atas nama Fery Firmansah dari Universitas Widya Dharma Klaten dengan judul The Odd Harmonious Labeling of Layered Graphs'. Below the text is a scanned receipt from BSI (Bank Syariah Indonesia) for a transfer of Rp 12.506.000,00 to account number 231235240837. A second email from 'JTAM Official <jtam.ummat@gmail.com>' dated 'Jum, 24 Mar, 14.25' is partially visible at the bottom. The Windows taskbar at the bottom shows the search bar, taskbar icons, and system tray with the date '10:18 04/07/2023'.

Sertifikat Publikasi Artikel

The screenshot displays a PDF document titled 'Sertifikat Fery Firmansah.pdf' opened in Google Docs. The certificate is for 'Author Certificate FKIP UMMat Scientific Journal' with 'Paper ID 12506'. It is awarded to 'Fery Firmansah' for the article 'The Odd Harmonious Labeling of Layered Graphs'. The text states: 'We hereby notify that the person above as Author of JTAM (Jurnal Teori dan Aplikasi Matematika), Accredited SINTA 2 in Volume 7 Issues 2 April 2023.' The certificate features logos for 'FKIP UMMat', 'SINTA', and 'Google Scholar'. It is signed by the 'Editorial Team' with a signature and stamp. The Windows taskbar at the bottom shows the search bar, taskbar icons, and system tray with the date '10:18 04/07/2023'.

Publikasi Artikel (6 April 2023)

The screenshot shows the journal's homepage for the article "The Odd Harmonious Labeling of Layered Graphs" by Fery Firmansah. The journal is "Jurnal Teori dan Aplikasi Matematika" (JTAM) from Universitas Muhammadiyah Mataram. The page includes a navigation menu, a sidebar with a "QUICK MENU" (Journal History, Editorial Team, Reviewer Team, Focus and Scope, Author Guidelines, Publication Ethics, Open Access Policy, Peer Review Process, Screening Plagiarism), and an abstract section. The abstract discusses the properties of odd harmonious labeling on layered graphs. A "Send message" button is visible in the bottom left. The browser's taskbar shows several open files related to the journal.

The screenshot displays the "References" section of the article. It lists several academic papers related to graph theory and harmonious labeling. A watermark "WE ARE HERE" is overlaid on the left side of the page. The right sidebar contains a "CONTACT US" section, a "SCOPUS CITATION ANALYSIS" section, an "ABOUT THE AUTHOR" section for Fery Firmansah, and a "DOWNLOAD DOCUMENT" section with links for "JTAM Template" and "Registration Form". The browser's taskbar shows the system time as 3:45 on 21/08/2024.

References:

- Abdel-Aal, M. E. (2013). Odd Harmonious Labelings of Cyclic Snakes. *International Journal on Applications of Graph Theory In Wireless Ad Hoc Networks And Sensor Networks*, 5(3), 1–11. <https://doi.org/10.5121/jgraphoc.2013.5301>
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The Odd Harmonious Labeling of Layered Graphs

ABSTRACT

Article History:

Received : D-M-20XX
 Revised : D-M-20XX
 Accepted : D-M-20XX
 Online : D-M-20XX

Keyword :

Harmonious labeling;
 Layared graphs;
 Odd harmonious
 graphs; Odd
 harmonious labeling;



<https://doi.org/10.31764/jtam.vXiY.ZZZ>



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A. INTRODUCTION

Graph labeling has been a highly developed graph theory topic in recent years, in addition to researchers interested in developing the theory, some have also found applications of graph labeling in communication network problems, data security, or cryptography. Graph labeling is basically labeling vertices and edges with specific properties (Gallian, 2019). There are several types of graph labeling, and one type of graph labeling studied by researchers is the odd harmonious labeling. The graph $G(p, q)$ with $p = |V(G)|$ and $q = |E(G)|$ is an odd harmonious graph if it fulfills the injective vertex labeling function $f: V(G) \rightarrow \{0, 1, 2, 3, \dots, 2q - 1\}$ and the bijective edge labeling function $f^*: E(G) \rightarrow \{1, 3, 5, 7, \dots, 2q - 1\}$ defined by $f^*(ab) = f(a) + f(b)$ (Liang & Bai, 2009).

Here are some odd harmonious graph classes that have been found by researchers. Abdel Al has obtained odd harmonious labeling of cyclic snake graphs (Abdel-Aal, 2013). Saputri et al have obtained dumbbell graphs are odd harmonious graphs (Saputri et al., 2013). Jeyanthi and Philo have proved that shadow graphs are cycles graphs with sharing a common vertex and edge are odd harmonious graphs (Jeyanthi & Philo, 2016). Abdel-Aal and Seoud have proved the odd harmonious labeling of splitting graphs (Abdel-Aal & Seoud, 2016). Firmansah have obtained odd harmonious graph classes, namely snake net graphs (Firmansah & Yuwono, 2017a) and amalgamation of double quadrilateral windmill graphs (Firmansah & Syaifuddin, 2018).

Renuka and Balaganesan have proved odd harmonious labeling of complete bipartite graphs (Renuka & Balaganesan, 2018a) and triangular snake graphs (Renuka & Balaganesan,

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 Ex: Jeyanthi, Firmansyah, Renuka, etc.

2018b). Seoud and Hafez are introducing strongly odd harmonious graphs (Seoud & Hafez, 2018). Kalaimathi and Balamurugan obtained computation of even odd harmonious labeling (Kalaimathi & Balamurugan, 2019). Jeyanthi and Philo have obtained odd harmonious labeling of pyramid graphs (Jeyanthi & Philo, 2019), line and disjoint union of graphs (Philo & Jeyanthi, 2021), and step ladder graphs are odd harmonious graphs (Jeyanthi & Philo, 2020). In another paper, Jeyanthi et al have proved that super subdivision graphs are odd harmonious graphs (Jeyanthi, Philo, & Siddiqui, 2019) and grid graphs are odd harmonious graphs (Jeyanthi, Philo, & Youssef, 2019).

Febriana and Sugeng have proved squid graphs and double squid graphs are odd harmonious graphs (Febriana & Sugeng, 2020). Govindarajan and Srividya have obtained even cycles graphs and dragons graphs are odd harmonious graphs (Govindarajan & Srividya, 2020). Furthermore, edge amalgamation from double quadrilateral graphs (Firmansah & Tasari, 2020), multiply net snake graphs (Firmansah, 2020b), and double triangular snake graphs (Senthil & Ganeshkumar, 2020). Firmansah and Giyarti have obtained an amalgamation of the generalized double quadrilateral windmill graph (Firmansah & Giyarti, 2021).

Zara et al have proved that even odd harmonious labeling of some graphs (Zala et al., 2021). Mumtaz and Silaban have obtained snake graphs with hair (Mumtaz & Silaban, 2021). In another paper, Mumtaz et al proved that matting graphs are odd harmonious graphs (Mumtaz et al., 2021). Sarasvati et al have obtained odd harmonious labeling of $P_n C_4$ and $P_n D_2(C_4)$ (Sarasvati et al., 2021). Firmansah has proved that string graphs are odd harmonious graphs (Firmansah, 2022). The relevant research results about odd harmonious graph classes that have been found can be seen in (Jeyanthi & Philo, 2015), (Jeyanthi et al., 2015), (Firmansah & Yuwono, 2017b), (Firmansah, 2017), (Sugeng et al., 2019), (Firmansah, 2020a) and (Pujawati et al., 2021).

In line with the relevant research results, the author constructs new graph classes, namely layered graphs $C(x, y)$ and layered graphs $D(x, y)$. Furthermore, the author has proved that the layered graph $C(x, y)$ and layered graph $D(x, y)$ satisfy the properties of odd harmonious labeling such that they are a new family of odd harmonious graphs. It is possible that this result can also be used to solve graph labeling problems, especially odd harmonious graph labeling.

B. METHODS

The research method used in this paper is a qualitative research method. The research flow consists of data collection, processing, and analysis. After the definition of the graph class is formed, it is continued with the vertex labeling construction and edge labeling construction. Furthermore, the construction of **toerem** and its proof are formed. The research method is as follows.

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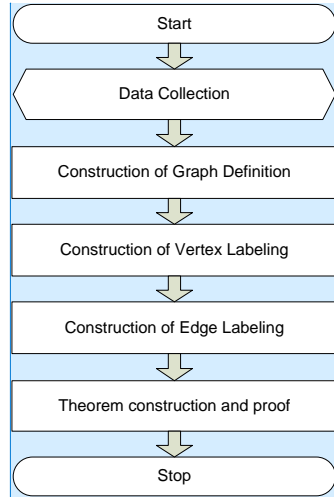


Figure 1. Flowchart research methodology

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C. RESULT AND DISCUSSION

1. Construction of layered graph $C(x, y)$ and its odd harmonious labeling

The following definition is given for a layered graph $C(x, y)$

Definition 1.

Layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is a graph with $V(C(x, y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y\}$ and $E(C(x, y)) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j-1} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j} a_i^j | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_{i-1}^{y+1} a_i^1 | 2 \leq i \leq x\}$.

In such a way that it is obtained $p = |V(C(x, y))| = 3xy + x$ and $q = |E(C(x, y))| = 4xy + x - 1$. The following is given the construction of the layered graph $C(x, y)$.

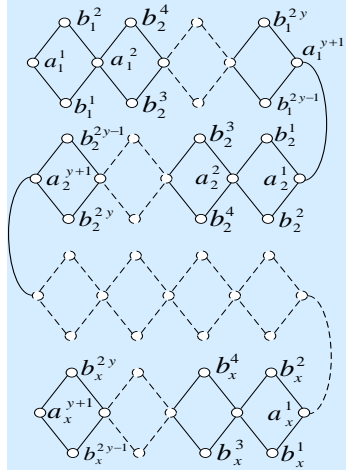


Figure 2. Construction of a layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$.

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 1. Selection data, explain the results
 2. Construction of Graph, explain the result
 and so on. You must consistently follow the method that you have laid out.

Theorem 2.

Layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph.

Proof.

Define vertex labeling function as follows

$$f(a_i^j) = (4y + 1)i + 4j - 4y - 5, 1 \leq i \leq x, 1 \leq j \leq y + 1 \tag{1}$$

$$f(b_i^j) = (4y + 1)i + 2j - 4y - 2, 1 \leq i \leq x, 1 \leq j \leq 2y \tag{2}$$

Based on (1) and (2), a different label is obtained at each vertex and $V(C(x, y)) \subseteq \{0, 1, 2, 3, \dots, 8xy + 2x - 3\}$ so the function f is injective.

Next, define edge labeling function as follows

$$f^*(a_i^j b_i^{2j-1}) = (8y + 2)i + 8j - 8y - 9, 1 \leq i \leq x, 1 \leq j \leq y \tag{3}$$

$$f^*(a_i^j b_i^{2j}) = (8y + 2)i + 8j - 8y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{4}$$

$$f^*(b_i^{2j-1} a_i^{j+1}) = (8y + 2)i + 8j - 8y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{5}$$

$$f^*(b_i^{2j} a_i^{j+1}) = (8y + 2)i + 8j - 8y - 3, 1 \leq i \leq x, 1 \leq j \leq y \tag{6}$$

$$f^*(a_{i-1}^{y+1} a_i^1) = (8y + 2)i - 8y - 3, 2 \leq i \leq x \tag{7}$$

Based on (3), (4), (5), (6) and (7) a different label is obtained at each edge and $E(C(x, y)) = \{1, 3, 5, 7, \dots, 8xy + 2x - 3\}$ so the function f^* is bijective.

Consequently the layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph ■

Here is the odd harmonious graph of the layered graph $C(5, 5)$.

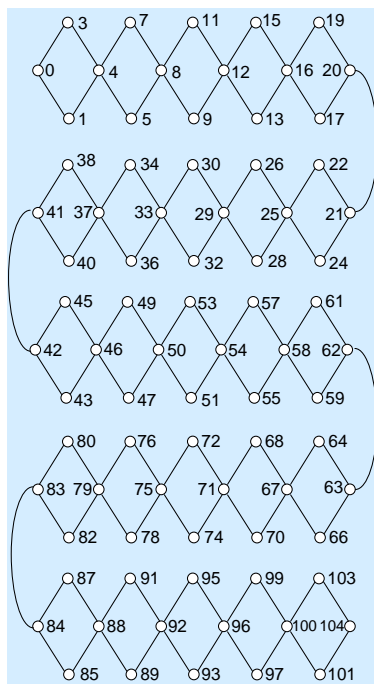


Figure 3. The layered graph $C(5, 5)$

2. Construction of layered graph $D(x, y)$ and its odd harmonious labeling

The following definition is given for a layered graph $D(x, y)$

Definition 3.

Layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$ is a graph with

$$V(D(x, y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y + 1\} \cup$$

$$\{c_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \text{ and } E(D(x, y)) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$$

$$\{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$$

$$\{b_i^{2j-1} c_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{c_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$$

$$\{b_i^{2j} c_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{c_{i-1}^{y+1} b_i^1 | 2 \leq i \leq x\}.$$

In such a way that it is obtained $p = |V(D(x, y))| = 4xy + 2x$ and $q = |E(D(x, y))| = 6xy + 2x - 1$.

The following is given the construction of the layered graph $D(x, y)$.

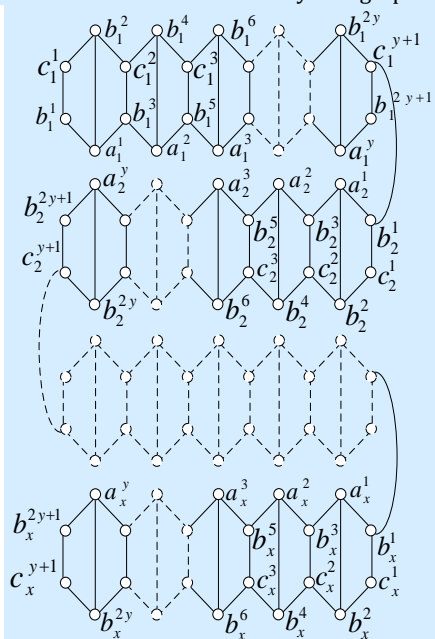


Figure 4. Construction of a layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$.

Theorem 23.

Layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph.

Proof.

Define vertex labeling function as follows

$$f(a_i^j) = (8y + 2)i + 2j - 8y - 4, 1 \leq i \leq x, 1 \leq j \leq y \tag{8}$$

$$f(b_i^j) = (4y + 2)i + 2j - 4y - 3, 1 \leq i \leq x, 1 \leq j \leq 2y + 1 \tag{9}$$

$$f(c_i^j) = (8y + 2)i + 2j - 2y - 4, 1 \leq i \leq x, 1 \leq j \leq y + 1 \tag{10}$$

Based on (8), (9) and (10) a different label is obtained at each vertex and $V(D(x, y)) \subseteq \{0, 1, 2, 3, \dots, 12xy + 4x - 3\}$ so the function f is injective.

Next, define edge labeling function as follows

$$f(a_i^j b_i^{2j-1}) = (12y + 4)i + 6j - 12y - 9, 1 \leq i \leq x, 1 \leq j \leq y \tag{11}$$

$$f(a_i^j b_i^{2j}) = (12y + 4)i + 6j - 12y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{12}$$

$$f(a_i^j b_i^{2j+1}) = (12y + 4)i + 6j - 12y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{13}$$

$$f(b_i^{2j-1} c_i^j) = (12y + 4)i + 6j - 6y - 9, 1 \leq i \leq x, 1 \leq j \leq y + 1 \tag{14}$$

$$f(b_i^{2j} c_i^j) = (12y + 4)i + 6j - 6y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{15}$$

$$f(b_i^{2j} c_i^{j+1}) = (12y + 4)i + 6j - 6y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{16}$$

$$f(c_{i-1}^{y+1} b_i^1) = (12y + 4)i - 12y - 5, 2 \leq i \leq x \tag{17}$$

Based on (11), (12), (13), (14), (15), (16) and (17) a different label is obtained at each edge and $E(D(x, y)) = \{1, 3, 5, 7, \dots, 12xy + 4x - 3\}$ so the function f^* is bijective.

Consequently the layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph ■

Here is the odd harmonious graph of the layered graph $D(5, 4)$.

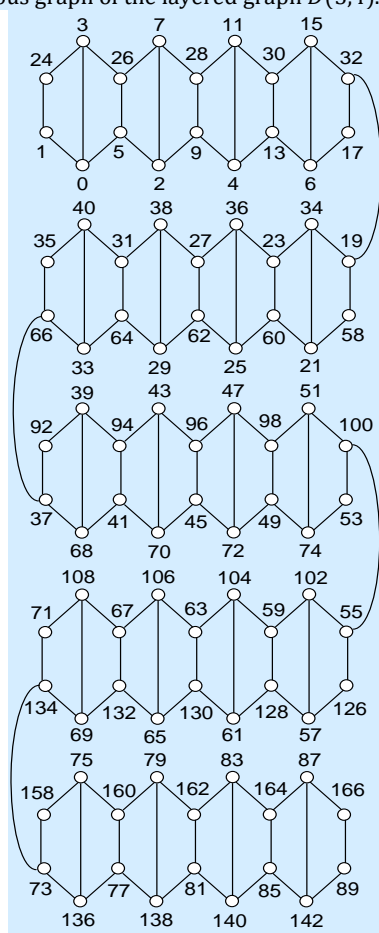


Figure 3. The layered graph $D(5, 4)$

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D. CONCLUSION AND SUGGESTIONS

Based on the results and discussion, a new graph class definition construction is obtained for the layered graphs $C(x, y)$ and layered graphs $D(x, y)$. Furthermore, it has been proven that layered graph $C(x, y)$ and layered graph $D(x, y)$ fulfill odd harmonious labeling so that they are odd harmonious graphs.

Suggestions for future research, this research can be continued by finding new graph classes that satisfy the properties of odd harmonious labeling.

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The Odd Harmonious Labeling of Layered Graphs

ABSTRACT

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Keyword :

Harmonious labeling;
 Layared graphs;
 Odd harmonious
 graphs; Odd
 harmonious labeling;



The research objective of this paper is to obtain odd harmonious labeling on layered graph $C(x,y)$ and layered graph $D(x,y)$. The research ~~method~~ used in this paper is a qualitative ~~research~~ method. The research flow consists of data collection, processing, and analysis. The research results ~~in this paper~~ show that the layered graph $C(x,y)$ and layered graph $D(x,y)$ fulfill odd harmonious labeling. Such that the layered graph $C(x,y)$ and layered graph $D(x,y)$ are odd harmonious graphs.

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<https://doi.org/10.31764/jtam.vXiY.ZZZ>



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A. INTRODUCTION

Graph labeling has been a highly developed graph theory topic in recent years, in addition to researchers interested in developing the theory, some have also found applications of graph labeling in communication network problems, data security, or cryptography. Graph labeling is basically labeling vertices and edges with specific properties (Gallian, 2019). There are several types of graph labeling, and one type of graph labeling studied by researchers is the odd harmonious labeling. The graph $G(p, q)$ with $p = |V(G)|$ and $q = |E(G)|$ is an odd harmonious graph if it fulfills the injective vertex labeling function $f: V(G) \rightarrow \{0, 1, 2, 3, \dots, 2q - 1\}$ and the bijective edge labeling function $f^*: E(G) \rightarrow \{1, 3, 5, 7, \dots, 2q - 1\}$ defined by $f^*(ab) = f(a) + f(b)$ (Liang & Bai, 2009).

Here are some odd harmonious graph classes that have been found by researchers. Abdel Al has obtained odd harmonious labeling of cyclic snake graphs (Abdel-Aal, 2013). Saputri et al have obtained dumbbell graphs are odd harmonious graphs (Saputri et al., 2013). Jeyanthi and Philo have proved that shadow graphs are cycles graphs with sharing a common vertex and edge are odd harmonious graphs (Jeyanthi & Philo, 2016). Abdel-Aal and Seoud have proved the odd harmonious labeling of splitting graphs (Abdel-Aal & Seoud, 2016). Firmansah have obtained odd harmonious graph classes, namely snake net graphs (Firmansah & Yuwono, 2017a) and amalgamation of double quadrilateral windmill graphs (Firmansah & Syaifuddin, 2018).

Renuka and Balaganesan (2018a) have proved odd harmonious labeling of complete bipartite graphs (Renuka & Balaganesan, 2018a) and triangular snake graphs (Renuka &

Balaganesan, 2018b). Seoud and Hafez are introducing strongly odd harmonious graphs (Seoud & Hafez, 2018). Kalaimathi and Balamurugan obtained computation of even odd harmonious labeling (Kalaimathi & Balamurugan, 2019). Jeyanthi and Philo have obtained odd harmonious labeling of pyramid graphs (Jeyanthi & Philo, 2019), line and disjoint union of graphs (Philo & Jeyanthi, 2021), and step ladder graphs are odd harmonious graphs (Jeyanthi & Philo, 2020). In another paper, Jeyanthi et al have proved that super subdivision graphs are odd harmonious graphs (Jeyanthi, Philo, & Siddiqui, 2019) and grid graphs are odd harmonious graphs (Jeyanthi, Philo, & Youssef, 2019).

Febriana and Sugeng (2020) have proved squid graphs and double squid graphs are odd harmonious graphs (Febriana & Sugeng, 2020). Govindarajan and Srividya have obtained even cycles graphs and dragons graphs are odd harmonious graphs (Govindarajan & Srividya, 2020). Furthermore, edge amalgamation from double quadrilateral graphs (Firmansah & Tasari, 2020), multiply net snake graphs (Firmansah, 2020b), and double triangular snake graphs (Senthil & Ganeshkumar, 2020). Firmansah and Giyarti have obtained an amalgamation of the generalized double quadrilateral windmill graph (Firmansah & Giyarti, 2021).

Zara et al (2021) have proved that even odd harmonious labeling of some graphs (Zara et al., 2021). Mumtaz and Silaban have obtained snake graphs with hair (Mumtaz & Silaban, 2021). In another paper, Mumtaz et al proved that matting graphs are odd harmonious graphs (Mumtaz et al., 2021). Sarasvati et al have obtained odd harmonious labeling of $P_n C_4$ and $P_n D_2(C_4)$ (Sarasvati et al., 2021). Firmansah has proved that string graphs are odd harmonious graphs (Firmansah, 2022). The relevant research results about odd harmonious graph classes that have been found can be seen in (Jeyanthi & Philo, 2015), (Jeyanthi et al., 2015), (Firmansah & Yuwono, 2017b), (Firmansah, 2017), (Sugeng et al., 2019), (Firmansah, 2020a) and (Pujiwati et al., 2021).

In line with the relevant research results, the author constructs new graph classes, namely layered graphs $C(x, y)$ and layered graphs $D(x, y)$. Furthermore, the author has proved that the layered graph $C(x, y)$ and layered graph $D(x, y)$ satisfy the properties of odd harmonious labeling such that they are a new family of odd harmonious graphs. It is possible that this result can also be used to solve graph labeling problems, especially odd harmonious graph labeling.

B. METHODS

The research method used in this paper is a qualitative research method. The research flow consists of data collection, processing, and analysis. After the definition of the graph class is formed, it is continued with the vertex labeling construction and edge labeling construction. Furthermore, the construction of theorem and its proof are formed. The research method is as follows.

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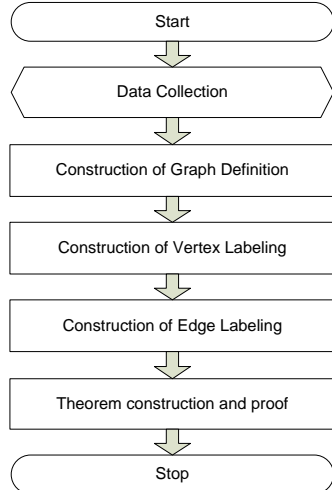


Figure 1. Flowchart research methodology

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C. RESULT AND DISCUSSION

1. Construction of layered graph $C(x, y)$ and its odd harmonious labeling

The following definition is given for a layered graph $C(x, y)$

Definition 1.

Layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is a graph with $V(C(x, y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y\}$ and $E(C(x, y)) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j-1} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_{i-1}^{y+1} a_i^1 | 2 \leq i \leq x\}$.

In such a way that it is obtained $p = |V(C(x, y))| = 3xy + x$ and $q = |E(C(x, y))| = 4xy + x - 1$. The following is given the construction of the layered graph $C(x, y)$.

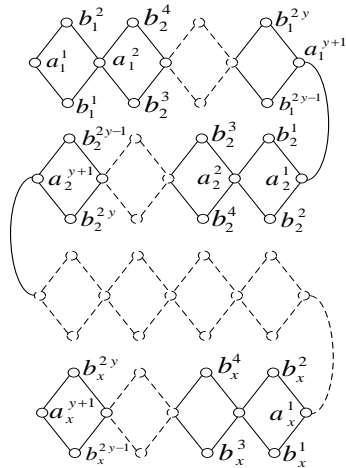


Figure 2. Construction of a layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$.

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Theorem 2.

Layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph.

Proof.

Define vertex labeling function as follows

$$f(a_i^j) = (4y + 1)i + 4j - 4y - 5, 1 \leq i \leq x, 1 \leq j \leq y + 1 \tag{1}$$

$$f(b_i^j) = (4y + 1)i + 2j - 4y - 2, 1 \leq i \leq x, 1 \leq j \leq 2y \tag{2}$$

Based on (1) and (2), a different label is obtained at each vertex and $V(C(x, y)) \subseteq \{0, 1, 2, 3, \dots, 8xy + 2x - 3\}$ so the function f is injective.

Next, define edge labeling function as follows

$$f^*(a_i^j b_i^{2j-1}) = (8y + 2)i + 8j - 8y - 9, 1 \leq i \leq x, 1 \leq j \leq y \tag{3}$$

$$f^*(a_i^j b_i^{2j}) = (8y + 2)i + 8j - 8y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{4}$$

$$f^*(b_i^{2j-1} a_i^{j+1}) = (8y + 2)i + 8j - 8y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{5}$$

$$f^*(b_i^{2j} a_i^{j+1}) = (8y + 2)i + 8j - 8y - 3, 1 \leq i \leq x, 1 \leq j \leq y \tag{6}$$

$$f^*(a_{i-1}^{y+1} a_i^1) = (8y + 2)i - 8y - 3, 2 \leq i \leq x \tag{7}$$

Based on (3), (4), (5), (6) and (7) a different label is obtained at each edge and $E(C(x, y)) = \{1, 3, 5, 7, \dots, 8xy + 2x - 3\}$ so the function f^* is bijective.

Consequently the layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph ■

Here is the odd harmonious graph of the layered graph $C(5, 5)$.

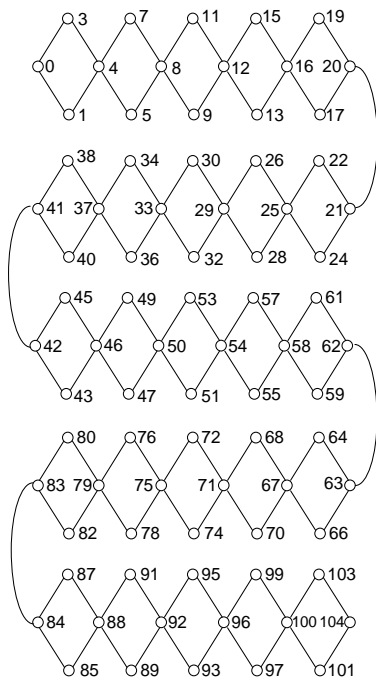


Figure 3. The layered graph $C(5, 5)$

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2. Construction of layered graph $D(x, y)$ and its odd harmonious labeling

The following definition is given for a layered graph $D(x, y)$

Definition 3.

Layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$ is a graph with

$$V(D(x, y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y + 1\} \cup$$

$$\{c_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \text{ and } E(D(x, y)) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$$

$$\{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$$

$$\{b_i^{2j-1} c_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{c_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$$

$$\{b_i^{2j} c_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{c_{i-1}^{y+1} b_i^1 | 2 \leq i \leq x\}.$$

In such a way that it is obtained $p = |V(D(x, y))| = 4xy + 2x$ and $q = |E(D(x, y))| = 6xy + 2x - 1$.

The following is given the construction of the layered graph $D(x, y)$.

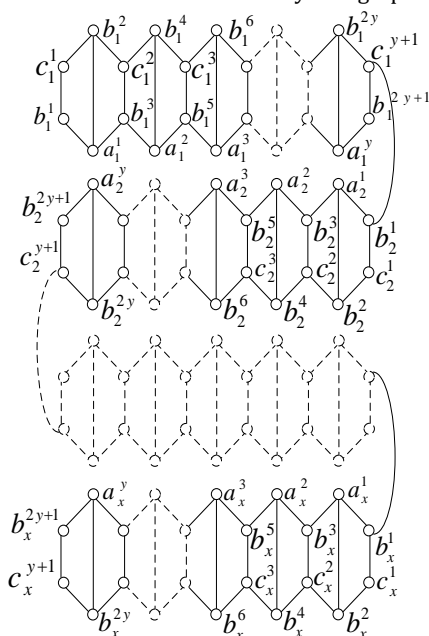


Figure 4. Construction of a layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$.

Commented [T10]: Please follow each figure with a detailed and clear description for easy understanding by the reader.

Theorem 2.

Layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph.

Proof.

Define vertex labeling function as follows

$$f(a_i^j) = (8y + 2)i + 2j - 8y - 4, 1 \leq i \leq x, 1 \leq j \leq y \tag{8}$$

$$f(b_i^j) = (4y + 2)i + 2j - 4y - 3, 1 \leq i \leq x, 1 \leq j \leq 2y + 1 \tag{9}$$

$$f(c_i^j) = (8y + 2)i + 2j - 2y - 4, 1 \leq i \leq x, 1 \leq j \leq y + 1 \tag{10}$$

Based on (8), (9) and (10) a different label is obtained at each vertex and $V(D(x, y)) \subseteq \{0, 1, 2, 3, \dots, 12xy + 4x - 3\}$ so the function f is injective.

Next, define edge labeling function as follows

$$f(a_i^j b_i^{2j-1}) = (12y + 4)i + 6j - 12y - 9, 1 \leq i \leq x, 1 \leq j \leq y \tag{11}$$

$$f(a_i^j b_i^{2j}) = (12y + 4)i + 6j - 12y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{12}$$

$$f(a_i^j b_i^{2j+1}) = (12y + 4)i + 6j - 12y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{13}$$

$$f(b_i^{2j-1} c_i^j) = (12y + 4)i + 6j - 6y - 9, 1 \leq i \leq x, 1 \leq j \leq y + 1 \tag{14}$$

$$f(c_i^j b_i^{2j}) = (12y + 4)i + 6j - 6y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{15}$$

$$f(b_i^{2j} c_i^{j+1}) = (12y + 4)i + 6j - 6y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{16}$$

$$f(c_{i-1}^{y+1} b_i^1) = (12y + 4)i - 12y - 5, 2 \leq i \leq x \tag{17}$$

Based on (11), (12), (13), (14), (15), (16) and (17) a different label is obtained at each edge and $E(D(x, y)) = \{1, 3, 5, 7, \dots, 12xy + 4x - 3\}$ so the function f^* is bijective.

Consequently the layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph ■

Here is the odd harmonious graph of the layered graph $D(5, 4)$.

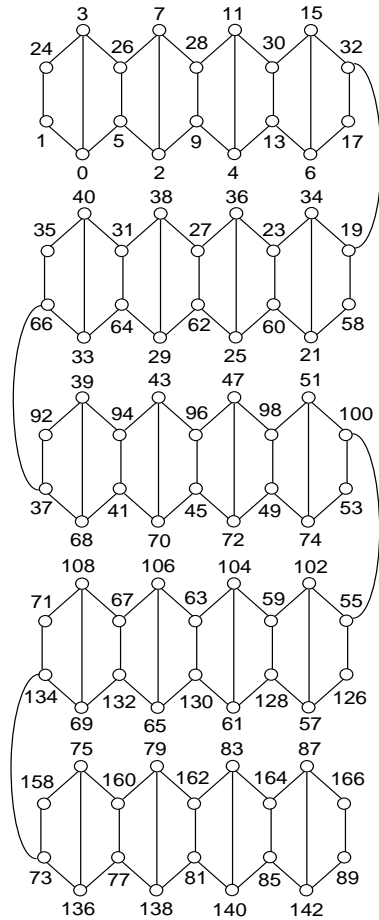


Figure 3. The layered graph $D(5, 4)$

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D. CONCLUSION AND SUGGESTIONS

Based on the results and discussion, a new graph class definition construction is obtained for the layered graphs $C(x, y)$ and layered graphs $D(x, y)$. Furthermore, it has been proven that layered graph $C(x, y)$ and layered graph $D(x, y)$ fulfill odd harmonious labeling so that they are odd harmonious graphs.

Suggestions for future research, this research can be continued by finding new graph classes that satisfy the properties of odd harmonious labeling.

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