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Title and Abstract

Title   The Odd Harmonious Labeling of Layered Graphs

Abstract   Graphs that have the properties of odd harmonious labeling are odd harmonious graphs. The research objective of this paper is to obtain odd harmonious labeling on layered graph C(x,y) and layered graph D(x,y). The research used in this paper is a qualitative method. The research flow consists of data collection, processing, and analysis. The data collection stage consists of constructing the definition of the new class graph, the data processing stage consists of constructing the vertex labeling and edge labeling, and the data analysis stage consists of constructing the theorem and proving it. The research results show that the layered graph C(x,y) and layered graph D(x,y) fulfill odd harmonious labeling. Such that the layered graph C(x,y) and layered graph D(x,y) are odd harmonious graphs. The benefit of this research is to add new properties of odd harmonious graphs. In addition, it does not rule out the possibility that this research can be developed again both in theory and application.

Indexing

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References

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The Odd Harmonious Labeling of Layered Graphs

ABSTRACT

The research objective of this paper is to obtain odd harmonious labeling on layered graph C(x,y) and layered graph D(x,y). The research method used in this paper is a qualitative research method. The research flow consists of data collection, processing, and analysis, The research results in this paper show that the layered graph C(x,y) and layered graph D(x,y) fulfill odd harmonious labeling. Such that the layered graph C(x,y) and layered graph D(x,y) are odd harmonious graphs.

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Figure 1. Flowchart research methodology

C. RESULT AND DISCUSSION

1. **Construction of layered graph $C(x, y)$ and its odd harmonious labeling**

The following definition is given for a layered graph $C(x, y)$

Definition 1.

Layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is a graph with $V(C(x,y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y\}$ and $E(C(x,y)) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j-1} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_{i-1}^{y+1} a_i^1 | 2 \leq i \leq x\}$.

In such a way that it is obtained $p = |V(C(x,y))| = 3xy + x$ and $q = |E(C(x,y))| = 4xy + x - 1$. The following is given the construction of the layered graph $C(x, y)$.

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The Odd Harmonious Labeling of Layered Graphs

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The research objective of this paper is to obtain odd harmonious labeling on layered graph C(x.y) and layered graph D(x.y). The research ~~method~~ used in this paper is a qualitative ~~research~~ method. The research flow consists of data collection, processing, and analysis. The research results ~~in this paper~~ show that the layered graph C(x,y) and layered graph D(x.y) fulfill odd harmonious labeling. Such that the layered graph C(x.y) and layered graph D(x.y) are odd harmonious graphs.

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D. CONCLUSION AND SUGGESTIONS

Based on the results and discussion, a new graph class definition construction is obtained for the layered graphs $C(x, y)$ and layered graphs $D(x, y)$. Furthermore, it has been proven that layered graph $C(x, y)$ and layered graph $D(x, y)$ fulfill odd harmonious labeling so that they are odd harmonious graphs.

Suggestions for future research, this research can be continued by finding new graph classes that satisfy the properties of odd harmonious labeling.

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The Odd Harmonious Labeling of Layered Graphs

Fery Firmansah

Abstract

Graphs that have the properties of odd harmonious labeling are odd harmonious graphs. The research objective of this paper is to obtain odd harmonious labeling on layered graph C(x,y) and layered graph D(x,y). The research used in this paper is a qualitative method. The research flow consists of data collection, processing, and analysis. The data collection stage consists of constructing the definition of the new class graph, the data processing stage consists of constructing the vertex labeling and edge labeling, and the data analysis stage consists of constructing the theorem and proving it. The research results show that the layered graph C(x,y) and layered graph D(x,y) fulfill odd harmonious labeling. Such that the layered graph C(x,y) and layered graph D(x,y) are odd harmonious graphs. The benefit of this research is to add new properties of odd harmonious graphs. In addition, it does not rule out the possibility that this research can be developed again both in theory and application.

Keywords

Harmonious labeling; Layered graphs; Odd harmonious graphs; Odd harmonious labelling.

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The Odd Harmonious Labeling of Layered Graphs

ABSTRACT

The research objective of this paper is to obtain odd harmonious labeling on layered graph C(x,y) and layered graph D(x,y). The research method used in this paper is a qualitative research method. The research flow consists of data collection, processing, and analysis. The research results in this paper show that the layered graph C(x,y) and layered graph D(x,y) fulfill odd harmonious labeling. Such that the layered graph C(x,y) and layered graph D(x,y) are odd harmonious graphs.

—————————— ◆ ——————————

A. INTRODUCTION

Graph labeling has been a highly developed graph theory topic in recent years, in addition to researchers interested in developing the theory, some have also found applications of graph labeling in communication network problems, data security, or cryptography. Graph labeling is basically labeling vertices and edges with specific properties (Gallian, 2019). There are several types of graph labeling, and one type of graph labeling studied by researchers is the odd harmonious labeling. The graph $G(p,q)$ with $p = |V(G)|$ and $q = |E(G)|$ is an odd harmonious graph if it fulfills the injective vertex labeling function $f: V(G) \rightarrow \{0,1,2,3,\dots,2q-1\}$ and the bijective edge labeling function $f^*: E(G) \rightarrow \{1,3,5,7,\dots,2q-1\}$ defined by $f^*(ab) = f(a) + f(b)$ (Liang & Bai, 2009).

Here are some odd harmonious graph classes that have been found by researchers. Abdel Al has obtained odd harmonious labeling of cyclic snake graphs (Abdel-Aal, 2013). Saputri et al have obtained dumbbell graphs are odd harmonious graphs (Saputri et al., 2013). Jeyanthi and Philo have proved that shadow graphs are cycles graphs with sharing a common vertex and edge are odd harmonious graphs (Jeyanthi & Philo, 2016). Abdel-Aal and Seoud have proved the odd harmonious labeling of splitting graphs (Abdel-Aal & Seoud, 2016). Firmansah have obtained odd harmonious graph classes, namely snake net graphs (Firmansah & Yuwono, 2017a) and amalgamation of double quadrilateral windmill graphs (Firmansah & Syaifuddin, 2018).

Renuka and Balaganesan have proved odd harmonious labeling of complete bipartite graphs (Renuka & Balaganesan, 2018a) and triangular snake graphs (Renuka & Balaganesan,

2018b). Seoud and Hafez are introducing strongly odd harmonious graphs (Seoud & Hafez, 2018). Kalaimathi and Balamurugan obtained computation of even odd harmonious labeling (Kalaimathi & Balamurugan, 2019). Jeyanthi and Philo have obtained odd harmonious labeling of pyramid graphs (Jeyanthi & Philo, 2019), line and disjoint union of graphs (Philo & Jeyanthi, 2021), and step ladder graphs are odd harmonious graphs (Jeyanthi & Philo, 2020). In another paper, Jeyanthi et al have proved that super subdivision graphs are odd harmonious graphs (Jeyanthi, Philo, & Siddiqui, 2019) and grid graphs are odd harmonious graphs (Jeyanthi, Philo, & Youssef, 2019).

Febriana and Sugeng have proved squid graphs and double squid graphs are odd harmonious graphs (Febriana & Sugeng, 2020). Govindarajan and Srividya have obtained even cycles graphs and dragons graphs are odd harmonious graphs (Govindarajan & Srividya, 2020). Furthermore, edge amalgamation from double quadrilateral graphs (Firmansah & Tasari, 2020), multiply net snake graphs (Firmansah, 2020b), and double triangular snake graphs (Senthil & Ganeshkumar, 2020). Firmansah and Giyarti have obtained an amalgamation of the generalized double quadrilateral windmill graph (Firmansah & Giyarti, 2021).

Zara et al have proved that even odd harmonious labeling of some graphs (Zala et al., 2021). Mumtaz and Silaban have obtained snake graphs with hair (Mumtaz & Silaban, 2021). In another paper, Mumtaz et all proved that matting graphs are odd harmonious graphs (Mumtaz et al., 2021). Sarasvati et al have obtained odd harmonious labeling of Pn C4 and Pn D2(C4) (Sarasvati et al., 2021). Firmansah has proved that string graphs are odd harmonious graphs (Firmansah, 2022). The relevant research results about odd harmonious graph classes that have been found can be seen in (Jeyanthi & Philo, 2015), (Jeyanthi et al., 2015), (Firmansah & Yuwono, 2017b), (Firmansah, 2017), (Sugeng et al., 2019), (Firmansah, 2020a) and (Pujiwati et al., 2021).

In line with the relevant research results, the author constructs new graph classes, namely layered graphs $C(x, y)$ and layered graphs $D(x, y)$. Furthermore, the author has proved that the layered graph $C(x, y)$ and layered graph $D(x, y)$ satisfy the properties of odd harmonious labeling such that they are a new family of odd harmonious graphs. It is possible that this result can also be used to solve graph labeling problems, especially odd harmonious graph labeling.

B. METHODS

The research method used in this paper is a qualitative research method. The research flow consists of data collection, processing, and analysis. After the definition of the graph class is formed, it is continued with the vertex labeling construction and edge labeling construction. Furthermore, the construction of toerem and its proof are formed. The research method is as follows.

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Figure 1. Flowchart research methodology

C. RESULT AND DISCUSSION

1. **Construction of layered graph $C(x, y)$ and its odd harmonious labeling**

The following definition is given for a layered graph $C(x, y)$

Definition 1.

Layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is a graph with $V\big(C(x, y)\big) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y\}$ and $E\big(C(x, y)\big) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j-1} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_{i-1}^{y+1} a_i^1 | 2 \leq i \leq x\}$.

In such a way that it is obtained $p = |V\big(C(x, y)\big)| = 3xy + x$ and $q = |E\big(C(x, y)\big)| = 4xy + x - 1$. The following is given the construction of the layered graph $C(x, y)$.

Figure 2. Construction of a layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$.

Theorem 2.
Layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph.
Proof.
  Define vertex labeling function as follows

$$f(a_i^j) = (4y + 1)i + 4j - 4y - 5, 1 \leq i \leq x, 1 \leq j \leq y + 1 \tag{1}$$
$$f(b_i^j) = (4y + 1)i + 2j - 4y - 2, 1 \leq i \leq x, 1 \leq j \leq 2y \tag{2}$$

  Based on (1) and (2), a different label is obtained at each vertex and $V(C(x, y)) \subseteq \{0, 1, 2, 3, \ldots, 8xy + 2x - 3\}$ so the function f is injective.
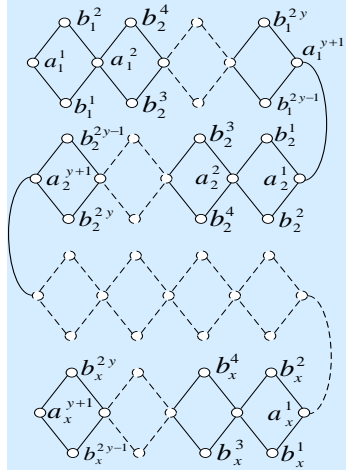 Next, define edge labeling function as follows

$$f^*\left(a_i^j b_i^{2j-1}\right) = (8y + 2)i + 8j - 8y - 9, 1 \leq i \leq x, 1 \leq j \leq y \tag{3}$$
$$f^*\left(a_i^j b_i^{2j}\right) = (8y + 2)i + 8j - 8y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{4}$$
$$f^*\left(b_i^{2j-1} a_i^{j+1}\right) = (8y + 2)i + 8j - 8y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{5}$$
$$f^*\left(b_i^{2j} a_i^{j+1}\right) = (8y + 2)i + 8j - 8y - 3, 1 \leq i \leq x, 1 \leq j \leq y \tag{6}$$
$$f^*\left(a_{i-1}^{y+1} a_i^1\right) = (8y + 2)i - 8y - 3, 2 \leq i \leq x \tag{7}$$

  Based on (3), (4), (5), (6) and (7) a different label is obtained at each edge and $E(C(x, y)) = \{1, 3, 5, 7, \ldots, 8xy + 2x - 3\}$ so the function f^* is bijective.
  Consequently the layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph ∎

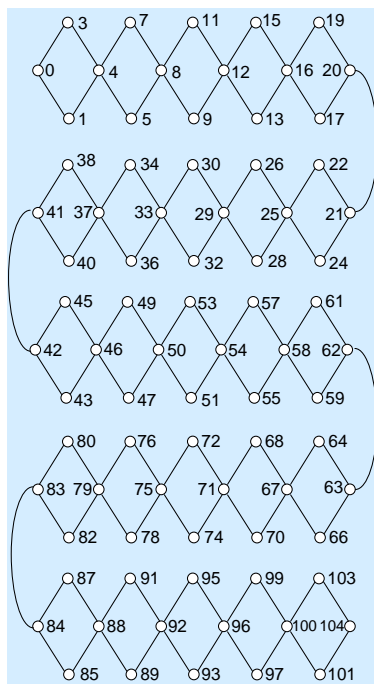Here is the odd harmonious graph of the layered graph $C(5,5)$.

Figure 3. The layered graph $C(5,5)$

2. Construction of layered graph $D(x, y)$ and its odd harmonious labeling

The following definition is given for a layered graph $D(x, y)$

Definition 3.

Layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$ is a graph with

$V(D(x, y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y + 1\} \cup$
$\{c_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\}$ and $E(D(x, y)) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$
$\{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$
$\{b_i^{2j-1} c_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{c_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$
$\{b_i^{2j} c_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{c_{i-1}^{y+1} b_i^1 | 2 \leq i \leq x\}.$

In such a way that it is obtained $p = |V(D(x, y))| = 4xy + 2x$ and $q = |E(D(x, y))| = 6xy + 2x - 1$.

The following is given the construction of the layered graph $D(x, y)$.

Figure 4. Construction of a layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$.

Theorem 2̶3.

Layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph.

Proof.

Define vertex labeling function as follows

$$f(a_i^j) = (8y + 2)i + 2j - 8y - 4, 1 \leq i \leq x, 1 \leq j \leq y \qquad (8)$$
$$f(b_i^j) = (4y + 2)i + 2j - 4y - 3, 1 \leq i \leq x, 1 \leq j \leq 2y + 1 \qquad (9)$$
$$f(c_i^j) = (8y + 2)i + 2j - 2y - 4, 1 \leq i \leq x, 1 \leq j \leq y + 1 \qquad (10)$$

Based on (8), (9) and (10) a different label is obtained at each vertex and $V(D(x, y)) \subseteq \{0, 1, 2, 3, \ldots, 12xy + 4x - 3\}$ so the function f is injective.

Next, define edge labeling function as follows

$$f(a_i^j b_i^{2j-1}) = (12y+4)i + 6j - 12y - 9, 1 \le i \le x, 1 \le j \le y \tag{11}$$

$$f(a_i^j b_i^{2j}) = (12y+4)i + 6j - 12y - 7, 1 \le i \le x, 1 \le j \le y \tag{12}$$

$$f(a_i^j b_i^{2j+1}) = (12y+4)i + 6j - 12y - 5, 1 \le i \le x, 1 \le j \le y \tag{13}$$

$$f(b_i^{2j-1} c_i^j) = (12y+4)i + 6j - 6y - 9, 1 \le i \le x, 1 \le j \le y + 1 \tag{14}$$

$$f(c_i^j b_i^{2j}) = (12y+4)i + 6j - 6y - 7, 1 \le i \le x, 1 \le j \le y \tag{15}$$

$$f(b_i^{2j} c_i^{j+1}) = (12y+4)i + 6j - 6y - 5, 1 \le i \le x, 1 \le j \le y \tag{16}$$

$$f(c_{i-1}^{y+1} b_i^1) = (12y+4)i - 12y - 5, 2 \le i \le x \tag{17}$$

Based on (11), (12), (13), (14), (15), (16) and (17) a different label is obtained at each edge and $E(D(x,y)) = \{1,3,5,7,\dots,12xy + 4x - 3\}$ so the function f^* is bijective.

Consequently the layered graph $D(x,y)$ with $x \ge 1$ and $y \ge 1$ is an odd harmonious graph ∎

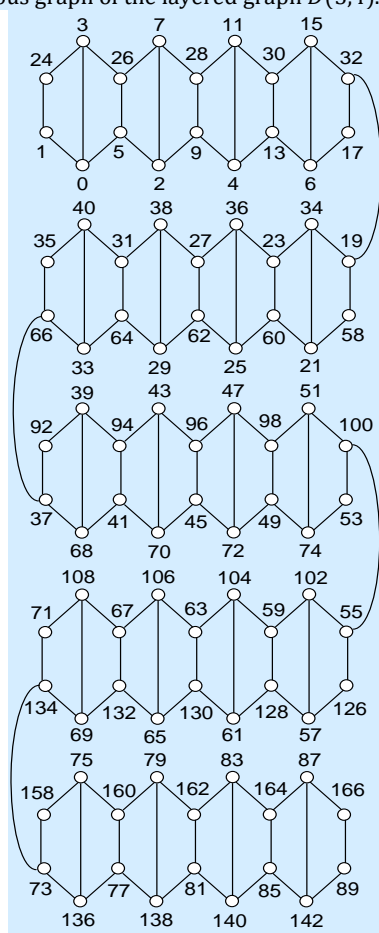Here is the odd harmonious graph of the layered graph $D(5,4)$.

Figure 3. The layered graph $D(5,4)$

D. CONCLUSION AND SUGGESTIONS

Based on the results and discussion, a new graph class definition construction is obtained for the layered graphs $C(x, y)$ and layered graphs $D(x, y)$. Furthermore, it has been proven that layered graph $C(x, y)$ and layered graph $D(x, y)$ fulfill odd harmonious labeling so that they are odd harmonious graphs.

Suggestions for future research, this research can be continued by finding new graph classes that satisfy the properties of odd harmonious labeling.

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The Odd Harmonious Labeling of Layered Graphs

ABSTRACT

The research objective of this paper is to obtain odd harmonious labeling on layered graph C(x,y) and layered graph D(x,y). The research method used in this paper is a qualitative research method. The research flow consists of data collection, processing, and analysis. The research results in this paper show that the layered graph C(x,y) and layered graph D(x,y) fulfill odd harmonious labeling. Such that the layered graph C(x,y) and layered graph D(x,y) are odd harmonious graphs.

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A. INTRODUCTION

Graph labeling has been a highly developed graph theory topic in recent years, in addition to researchers interested in developing the theory, some have also found applications of graph labeling in communication network problems, data security, or cryptography. Graph labeling is basically labeling vertices and edges with specific properties (Gallian, 2019). There are several types of graph labeling, and one type of graph labeling studied by researchers is the odd harmonious labeling. The graph $G(p,q)$ with $p = |V(G)|$ and $q = |E(G)|$ is an odd harmonious graph if it fulfills the injective vertex labeling function $f: V(G) \rightarrow \{0,1,2,3,\ldots,2q-1\}$ and the bijective edge labeling function $f^*: E(G) \rightarrow \{1,3,5,7,\ldots,2q-1\}$ defined by $f^*(ab) = f(a) + f(b)$ (Liang & Bai, 2009).

Here are some odd harmonious graph classes that have been found by researchers. Abdel Al has obtained odd harmonious labeling of cyclic snake graphs (Abdel-Aal, 2013). Saputri et al have obtained dumbbell graphs are odd harmonious graphs (Saputri et al., 2013). Jeyanthi and Philo have proved that shadow graphs are cycles graphs with sharing a common vertex and edge are odd harmonious graphs (Jeyanthi & Philo, 2016). Abdel-Aal and Seoud have proved the odd harmonious labeling of splitting graphs (Abdel-Aal & Seoud, 2016). Firmansah have obtained odd harmonious graph classes, namely snake net graphs (Firmansah & Yuwono, 2017a) and amalgamation of double quadrilateral windmill graphs (Firmansah & Syaifuddin, 2018).

Renuka and Balaganesan (2018a) have proved odd harmonious labeling of complete bipartite graphs (Renuka & Balaganesan, 2018a) and triangular snake graphs (Renuka &

Balaganesan, 2018b). Seoud and Hafez are introducing strongly odd harmonious graphs (Seoud & Hafez, 2018). Kalaimathi and Balamurugan obtained computation of even odd harmonious labeling (Kalaimathi & Balamurugan, 2019). Jeyanthi and Philo have obtained odd harmonious labeling of pyramid graphs (Jeyanthi & Philo, 2019), line and disjoint union of graphs (Philo & Jeyanthi, 2021), and step ladder graphs are odd harmonious graphs (Jeyanthi & Philo, 2020). In another paper, Jeyanthi et al have proved that super subdivision graphs are odd harmonious graphs (Jeyanthi, Philo, & Siddiqui, 2019) and grid graphs are odd harmonious graphs (Jeyanthi, Philo, & Youssef, 2019).

Febriana and Sugeng (2020) have proved squid graphs and double squid graphs are odd harmonious graphs (Febriana & Sugeng, 2020). Govindarajan and Srividya have obtained even cycles graphs and dragons graphs are odd harmonious graphs (Govindarajan & Srividya, 2020). Furthermore, edge amalgamation from double quadrilateral graphs (Firmansah & Tasari, 2020), multiply net snake graphs (Firmansah, 2020b), and double triangular snake graphs (Senthil & Ganeshkumar, 2020). Firmansah and Giyarti have obtained an amalgamation of the generalized double quadrilateral windmill graph (Firmansah & Giyarti, 2021).

Zara et al (2021) have proved that even odd harmonious labeling of some graphs (Zala et al., 2021). Mumtaz and Silaban have obtained snake graphs with hair (Mumtaz & Silaban, 2021). In another paper, Mumtaz et all proved that matting graphs are odd harmonious graphs (Mumtaz et al., 2021). Sarasvati et al have obtained odd harmonious labeling of Pn C4 and Pn D2(C4) (Sarasvati et al., 2021). Firmansah has proved that string graphs are odd harmonious graphs (Firmansah, 2022). The relevant research results about odd harmonious graph classes that have been found can be seen in (Jeyanthi & Philo, 2015), (Jeyanthi et al., 2015), (Firmansah & Yuwono, 2017b), (Firmansah, 2017), (Sugeng et al., 2019), (Firmansah, 2020a) and (Pujiwati et al., 2021).

In line with the relevant research results, the author constructs new graph classes, namely layered graphs $C(x, y)$ and layered graphs $D(x, y)$. Furthermore, the author has proved that the layered graph $C(x, y)$ and layered graph $D(x, y)$ satisfy the properties of odd harmonious labeling such that they are a new family of odd harmonious graphs. It is possible that this result can also be used to solve graph labeling problems, especially odd harmonious graph labeling.

B. METHODS

The research method used in this paper is a qualitative research method. The research flow consists of data collection, processing, and analysis. After the definition of the graph class is formed, it is continued with the vertex labeling construction and edge labeling construction. Furthermore, the construction of toerem and its proof are formed. The research method is as follows.

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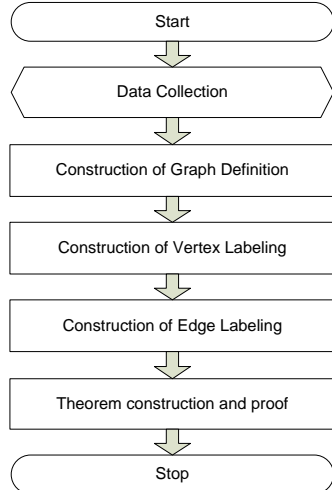
Figure 1. Flowchart research methodology

C. RESULT AND DISCUSSION

1. Construction of layered graph $C(x, y)$ and its odd harmonious labeling

The following definition is given for a layered graph $C(x, y)$

Definition 1.

Layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$ is a graph with $V(C(x, y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y\}$ and $E(C(x, y)) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j-1} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^{2j} a_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_{i-1}^{y+1} a_i^1 | 2 \leq i \leq x\}$.

In such a way that it is obtained $p = |V(C(x, y))| = 3xy + x$ and $q = |E(C(x, y))| = 4xy + x - 1$. The following is given the construction of the layered graph $C(x, y)$.
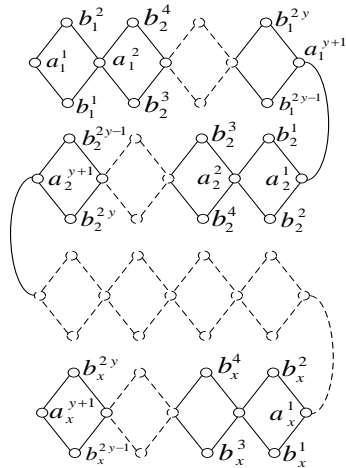

Figure 2. Construction of a layered graph $C(x, y)$ with $x \geq 1$ and $y \geq 1$.

Theorem 2.
Layered graph $C(x,y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph.
Proof.
Define vertex labeling function as follows

$$f(a_i^j) = (4y+1)i + 4j - 4y - 5, 1 \leq i \leq x, 1 \leq j \leq y+1 \tag{1}$$
$$f(b_i^j) = (4y+1)i + 2j - 4y - 2, 1 \leq i \leq x, 1 \leq j \leq 2y \tag{2}$$

Based on (1) and (2), a different label is obtained at each vertex and $V(C(x,y)) \subseteq \{0,1,2,3,\ldots,8xy + 2x - 3\}$ so the function f is injective.

Next, define edge labeling function as follows

$$f^*(a_i^j b_i^{2j-1}) = (8y+2)i + 8j - 8y - 9, 1 \leq i \leq x, 1 \leq j \leq y \tag{3}$$
$$f^*(a_i^j b_i^{2j}) = (8y+2)i + 8j - 8y - 7, 1 \leq i \leq x, 1 \leq j \leq y \tag{4}$$
$$f^*(b_i^{2j-1} a_i^{j+1}) = (8y+2)i + 8j - 8y - 5, 1 \leq i \leq x, 1 \leq j \leq y \tag{5}$$
$$f^*(b_i^{2j} a_i^{j+1}) = (8y+2)i + 8j - 8y - 3, 1 \leq i \leq x, 1 \leq j \leq y \tag{6}$$
$$f^*(a_{i-1}^{y+1} a_i^1) = (8y+2)i - 8y - 3, 2 \leq i \leq x \tag{7}$$

Based on (3), (4), (5), (6) and (7) a different label is obtained at each edge and $E(C(x,y)) = \{1,3,5,7,\ldots,8xy + 2x - 3\}$ so the function f^* is bijective.

Consequently the layered graph $C(x,y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph ∎

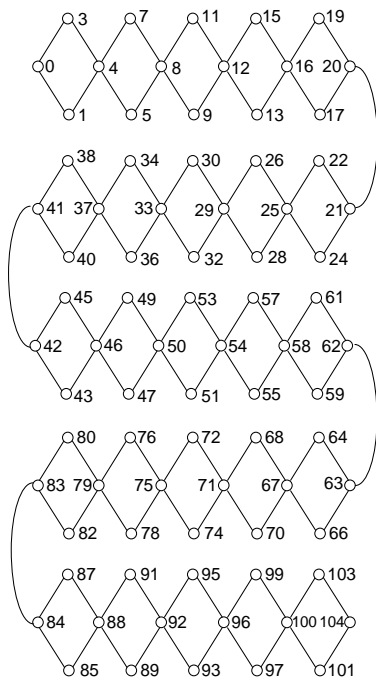Here is the odd harmonious graph of the layered graph $C(5,5)$.

Figure 3. The layered graph $C(5,5)$

2. Construction of layered graph $D(x, y)$ and its odd harmonious labeling

The following definition is given for a layered graph $D(x, y)$

Definition 3.

Layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$ is a graph with
$V(D(x, y)) = \{a_i^j | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{b_i^j | 1 \leq i \leq x, 1 \leq j \leq 2y + 1\} \cup$
$\{c_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\}$ and $E(D(x, y)) = \{a_i^j b_i^{2j-1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$
$\{a_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{a_i^j b_i^{2j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$
$\{b_i^{2j-1} c_i^j | 1 \leq i \leq x, 1 \leq j \leq y + 1\} \cup \{c_i^j b_i^{2j} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup$
$\{b_i^{2j} c_i^{j+1} | 1 \leq i \leq x, 1 \leq j \leq y\} \cup \{c_{i-1}^{y+1} b_i^1 | 2 \leq i \leq x\}$.

In such a way that it is obtained $p = |V(D(x, y))| = 4xy + 2x$ and $q = |E(D(x, y))| = 6xy + 2x - 1$.

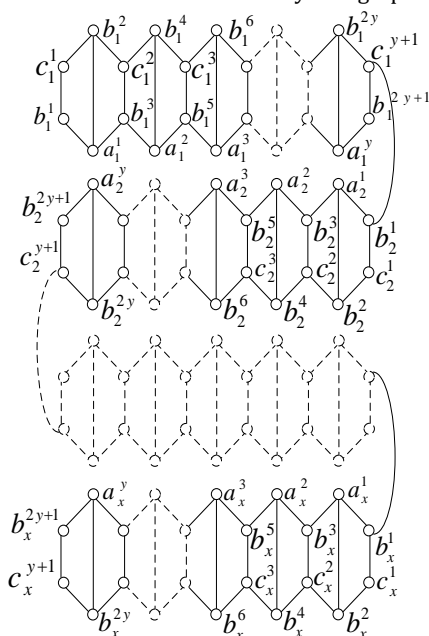The following is given the construction of the layered graph $D(x, y)$.

Figure 4. Construction of a layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$.

Theorem 2.

Layered graph $D(x, y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph.

Proof.

Define vertex labeling function as follows

$$f(a_i^j) = (8y + 2)i + 2j - 8y - 4, 1 \leq i \leq x, 1 \leq j \leq y \qquad (8)$$

$$f(b_i^j) = (4y + 2)i + 2j - 4y - 3, 1 \leq i \leq x, 1 \leq j \leq 2y + 1 \qquad (9)$$

$$f(c_i^j) = (8y + 2)i + 2j - 2y - 4, 1 \leq i \leq x, 1 \leq j \leq y + 1 \qquad (10)$$

Based on (8), (9) and (10) a different label is obtained at each vertex and $V(D(x, y)) \subseteq \{0, 1, 2, 3, \ldots, 12xy + 4x - 3\}$ so the function f is injective.

Next, define edge labeling function as follows

$$f\left(a_i^j b_i^{2j-1}\right) = (12y+4)i + 6j - 12y - 9, 1 \leq i \leq x, 1 \leq j \leq y \qquad (11)$$
$$f\left(a_i^j b_i^{2j}\right) = (12y+4)i + 6j - 12y - 7, 1 \leq i \leq x, 1 \leq j \leq y \qquad (12)$$
$$f\left(a_i^j b_i^{2j+1}\right) = (12y+4)i + 6j - 12y - 5, 1 \leq i \leq x, 1 \leq j \leq y \qquad (13)$$
$$f\left(b_i^{2j-1} c_i^j\right) = (12y+4)i + 6j - 6y - 9, 1 \leq i \leq x, 1 \leq j \leq y + 1 \qquad (14)$$
$$f\left(c_i^j b_i^{2j}\right) = (12y+4)i + 6j - 6y - 7, 1 \leq i \leq x, 1 \leq j \leq y \qquad (15)$$
$$f\left(b_i^{2j} c_i^{j+1}\right) = (12y+4)i + 6j - 6y - 5, 1 \leq i \leq x, 1 \leq j \leq y \qquad (16)$$
$$f\left(c_{i-1}^{y+1} b_i^1\right) = (12y+4)i - 12y - 5, 2 \leq i \leq x \qquad (17)$$

Based on (11), (12), (13), (14), (15), (16) and (17) a different label is obtained at each edge and $E\left(D(x,y)\right) = \{1,3,5,7,\ldots,12xy + 4x - 3\}$ so the function f^* is bijective.

Consequently the layered graph $D(x,y)$ with $x \geq 1$ and $y \geq 1$ is an odd harmonious graph ∎

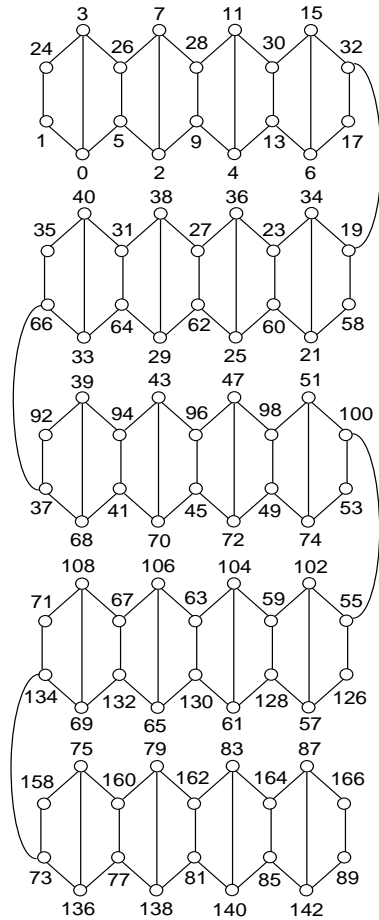Here is the odd harmonious graph of the layered graph $D(5,4)$.

Figure 3. The layered graph $D(5,4)$

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D. CONCLUSION AND SUGGESTIONS

Based on the results and discussion, a new graph class definition construction is obtained for the layered graphs $C(x, y)$ and layered graphs $D(x, y)$. Furthermore, it has been proven that layered graph $C(x, y)$ and layered graph $D(x, y)$ fulfill odd harmonious labeling so that they are odd harmonious graphs.

Suggestions for future research, this research can be continued by finding new graph classes that satisfy the properties of odd harmonious labeling.

ACKNOWLEDGEMENT

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