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Application of integer linear program in optimizing convection sector production results using branch and bound method

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Abstract

This study aimed to determine the application of the integer program in optimizing the production of the convection sector. Integer linear programming is a special form of linear programming in which the decision variable solutions are integers. Ayyumnah store as one part of the convection sectors with a home-scale does not have an appropriate strategy to optimize profits with limited materials owned. The method used in this study is an integer program with the branch and bound method. The result of this research is the optimal amount of production of long shirts and tunics at the Ayyumnah Store with maximum profit.

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INTRODUCTION

One type of linear program that is widely applied in common is integer linear programming. It is basically a special case of linear programming that the decision variable gained is an integer. One of the methods used to solve integer programming is the branch and bound method. The method has a working principle in the form of branching questions that do not have a solution with integer value (Siang, 2011).

The underneath research results are relevant and related to the application of integer linear programming using the branch and bound method. Bellenguez-Morineau and Néron, in 2007, applied the branch and bound method to solve the scheduling problem (Bellenguez-Morineau & Néron, 2007). Sharma, in 2012, solved the transportation problem using the integer program (Sharma, 2012). Liu and Gao introduced the quadratic integer program using the branch and bound method (Liu & Gao, 2015). Chiang and Wang, in 2015, implemented the branch and bounds method to solve a non-linear integer program (Chiang & Wang, 2015). Firmansah and Wulandari, in 2021, applied Integer linier programming in production results optimization using cutting plane method (Firmansah & Wulandari, 2021).

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Hernawati, in 2017, applied integer linear program to production optimization (Hernawati, 2017). Jannah, in 2018, utilized the branch and bound method to optimize the production of tempeh and tofu (Jannah A.M et al., 2018). Purba and Ahyaningsih, in 2020, also implemented an integer program with branch and bound methods on bread production (Purba & Ahyaningsih, 2020). While Buyung and Suhendar, in 2020, used the branch and bound method to optimize profits (Buyung & Suhendar, 2020). In another article, Buyung and Suhendar, in 2021, implemented an integer program with a branch and bound algorithm using QM for Windows. (Buyung & Suhendar, 2021).

Ayyumnah store as a part of the convection sector with a home industry scale does not have the proper strategy to optimize profits with limited raw materials. Those are the limited supply of raw materials for cotton and ceruti clothes to make long shirts and tunics. In line with related researches and solving real problems, the authors have succeeded in implementing integer linear programming using the branch and bound method to optimize the production of the convection sector. The results of this study prove the renewal of the application of mathematics in solving real problems.

METHOD

This research is a form of applied research that aims to provide an optimal solution to a problem. Primary research data is data obtained directly from the research location, namely the Ayyumnah Store in Krandon, Kwaren Village, Ngawen, Klaten Regency. The results of the Ayyumnah store's production process are influenced by the limited supply of raw materials for cotton and ceruti clothes to make long shirts and tunics. The daily availability of materials in this shop is 21 meters of cotton cloth and 13 meters of ceruti cloth. The need for long shirt products is 3 meters of cotton cloth and 2 meters of ceruti cloth with a profit of Rp. 40,000.00/pcs. Meanwhile, the need for tunic products is 2 meters of cotton cloth and 1 meter of ceruti cloth with a profit of Rp. 30,000.00/pcs. The stages of data analysis used in the research are by changing the data obtained into the form of an integer linear programming model and then finishing them with the branch and bound method. The decision variable must be an integer, for that the Ayyumnah store can obtain the maximum profit.

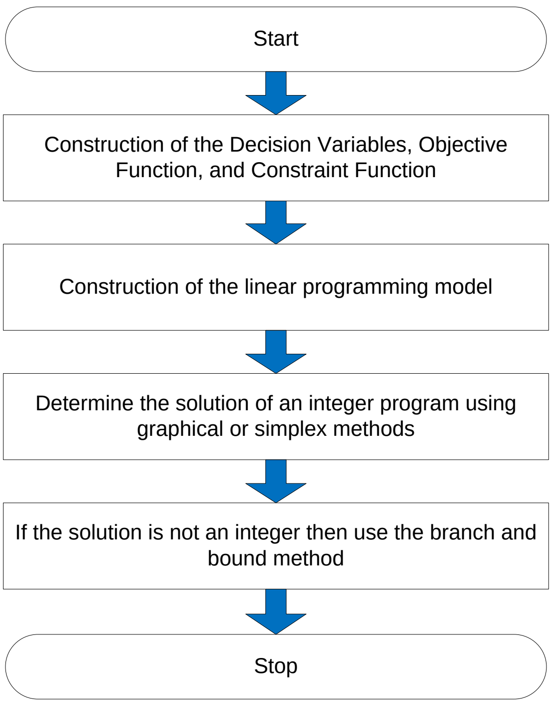

Figure 1. Flowchart of research methods

RESULTS AND DISCUSSION

Formation of integer linear programming model.

The Decision Variables

The products that will be optimized are long shirts and tunics produced by Ayyumnah shop. The decision variable is in the form of:

x_1 = number of long shirts produced/day (pcs)
x_2 = number of tunics produced/day (pcs)

The Objective Function

The goal of the research is to maximize the profit from the production of Ayyummah's long shirts and tunics.

Table 1. Production Profit

Clothing Type	Profit
Long Shirt	Rp. 40.000,00
Tunic	Rp. 30.000,00

Based on Table 1, the objective function is obtained as follows:
Maximize $\boldsymbol{z} = 40.000x_1 + 30.000x_2$ or Maximize $z = 4x_1 + 3x_2$ (in tens of thousands)

The Constraint Function

Ayyumnah store has obstacles in the form of limited supplies of raw materials for cotton and ceruti as shown in Table 2 as follows:

Table 2. Cloth Needs and Availability

Clothing Type	Cloth Raw Material	
	Catton (meter)	Ceruti (meter)
Long Shirt	3	2
Tunic	2	1
Availability	21	13

Based on Table 2, the constraint function can be formulated as follows:
Cotton cloth: $3x_1 + 2x_2 \leq 21$
Ceruti cloth: $2x_1 + x_2 \leq 13$
As of the general form of the linear integer program is obtained as follows.

Integer Linear Programming Model

Maximize :
$z = 4x_1 + 3x_2$
Constraint :
$3x_1 + 2x_2 \leq 21$
$2x_1 + x_2 \leq 13$
$x_1, x_2 \geq 0$ non negative integer
Solution integer linear programming problems using branch and bound method.

Solution with branch and bound method

Program-1
Maximize :
$z = 4x_1 + 3x_2$
Constraint :
$3x_1 + 2x_2 \leq 21$
$2x_1 + x_2 \leq 13$
$x_1, x_2 \geq 0$ non negative integer
Based on Figure 1, it is obtained:
Value for $z = 4x_1 + 3x_2$
$(0,0) \rightarrow z = 4.0 + 3.0 = 0$
$\left(0, \frac{21}{2}\right) \rightarrow z = 4.0 + 3.\frac{21}{2} = 31,5$
$\left(\frac{13}{2}, 0\right) \rightarrow z = 4.\frac{13}{2} + 3.0 = 26$
$(5,3) \rightarrow z = 4.5 + 3.3 = 29$

Therefore, the maximum point is found at $x_1 = 0$ dan $x_2 = \frac{21}{5} = 10{,}5$ with function value $z_1{}^* = 31{,}5$ (in tens thousand). Because x_2 is not an integer then the branching process must be executed by adding constraints $x_2 \leq 10$ on Program-2 dan $x_2 \geq 11$ on Program-3.

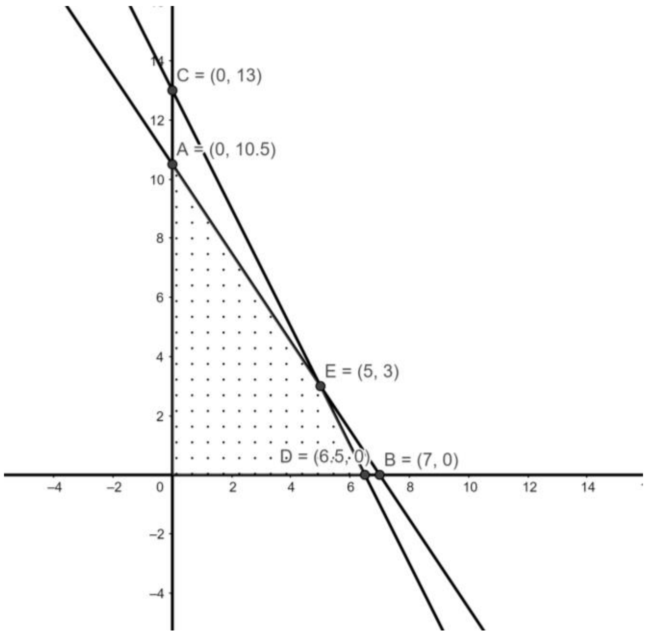

Figure 1. Program-1

Program-2
(Program-1 + constraint $x_2 \leq 10$)
Maximize :
$z = 4x_1 + 3x_2$
Constraint:
$3x_1 + 2x_2 \leq 21$
$2x_1 + x_2 \leq 13$
$x_2 \leq 10$
$x_1, x_2 \geq 0$ non negative integer
Based on Figure 2, it is obtained
Value for $z = 4x_1 + 3x_2$
$(0,0) \rightarrow z = 4.0 + 3.0 = 0$
$(0,10) \rightarrow z = 4.0 + 3.10 = 30$
$\left(\frac{1}{3}, 10\right) \rightarrow z = 4.\frac{1}{3} + 3.10 = 31{,}33$
$\left(\frac{13}{2}, 0\right) \rightarrow z = 4.\frac{13}{2} + 3.0 = 26$
$(5,3) \rightarrow z = 4.5 + 3.3 = 29$

Hence, the maximum point is found at $x_1 = \frac{1}{3}$ dan $x_2 = 10$ with function value $z_2{}^* = 31{,}33$. Program-2 completion is not an integer so it's not a solution.

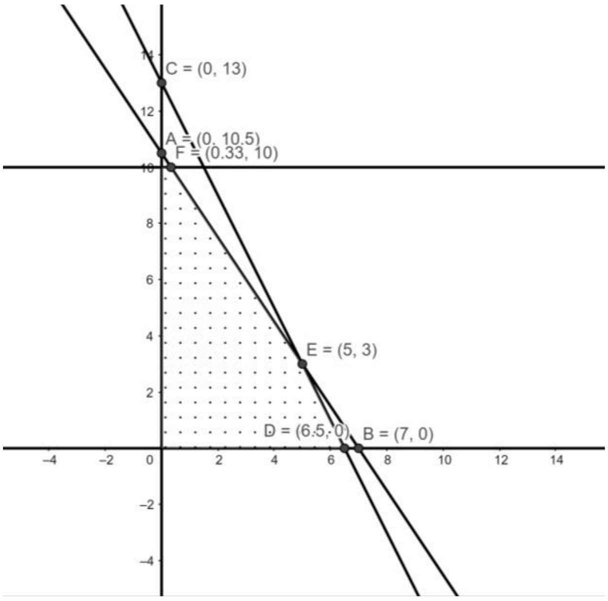

Figure 2. Program-2

Program-3
(Program-1 + constraint $x_2 \geq 11$)
Maximize :
$z = 4x_1 + 3x_2$
Constraints:
$3x_1 + 2x_2 \leq 21$
$2x_1 + x_2 \leq 13$
$x_2 \geq 11$
$x_1, x_2 \geq 0$ non negative integer

According to Figure 3, it is achived that Program-3 is not a feasible problem then it does not have an optimal solution. Program-3 is ignored and Program-2 is the only one that has to be processed further. On the other hand, let's pay attention to Program-2. Because x_1 is not an integer so the branching process is implemented by adding constraints $x_1 \leq 0$ on Program-4 and $x_1 \geq 1$ on Program-5.

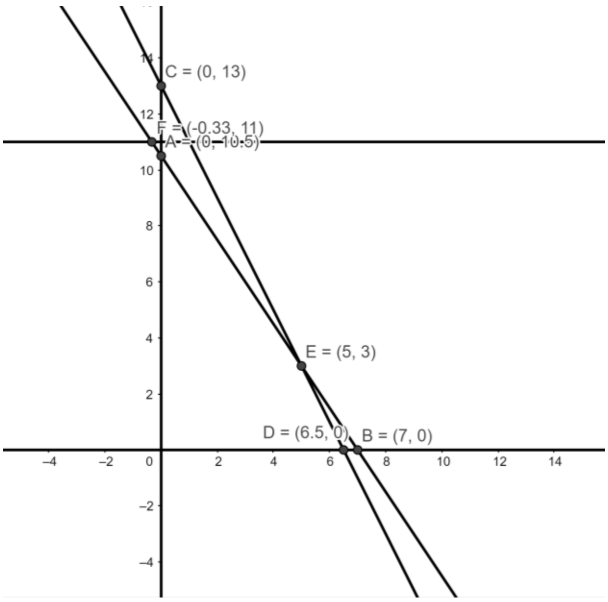

Figure 3. Program-3

Program-4
(Program-2 + constraint $x_1 \leq 0$)
Maximize :
$z = 4x_1 + 3x_2$

Constraints:
$3x_1 + 2x_2 \leq 21$
$2x_1 + x_2 \leq 13$
$x_1 \leq 0$
$x_1, x_2 \geq 0$ non negative integer
With the constraint of $x_1 \leq 0$, the conditions are a non negative integer. It means that $x_1 = 0$ Built upon Figure 4, it is obtained that:
Value for $z = 4x_1 + 3x_2$
$(0,0) \rightarrow z = 4.0 + 3.0 = 0$
$(0,10) \rightarrow z = 4.0 + 3.10 = 30$
With the result, the maximum point is situated at $x_1 = 0$ dan $x_2 = 10$ by the function value $z_4{}^* = 30$ (in ten thousand). The completion of Program-4 is already an integer.

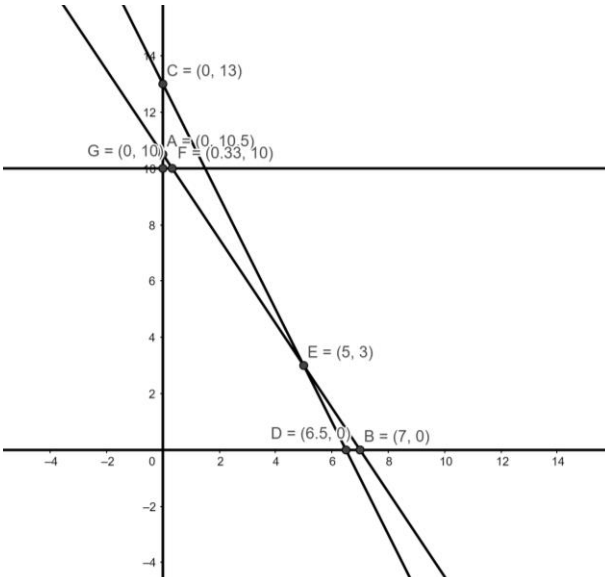

Figure 4. Program-4

Program-5
(Program-2 + constraint $x_1 \geq 1$)
Maximize:
$z = 4x_1 + 3x_2$
Constraints:
$3x_1 + 2x_2 \leq 21$
$2x_1 + x_2 \leq 13$
$x_1 \geq 1$
$x_1, x_2 \geq 0$ non negatif integer
According to Figure 5, it is gained that:
Value for $z = 4x_1 + 3x_2$
$(1,0) \rightarrow z = 4.1 + 3.0 = 1$
$(1,9) \rightarrow z = 4.1 + 3.9 = 31$
$\left(\frac{13}{2}, 0\right) \rightarrow z = 4.\frac{13}{2} + 3.0 = 26$
$(5,3) \rightarrow z = 4.5 + 3.3 = 29$
Consequently, the maximum point is found at $x_1 = 1$ and $x_2 = 9$ with the value function $z_5{}^* = 31$. The solution of Program-5 is already an integer with a function value is greater than the value of the Program-4 function. For that, the completion of Program-5 is the optimal solution.

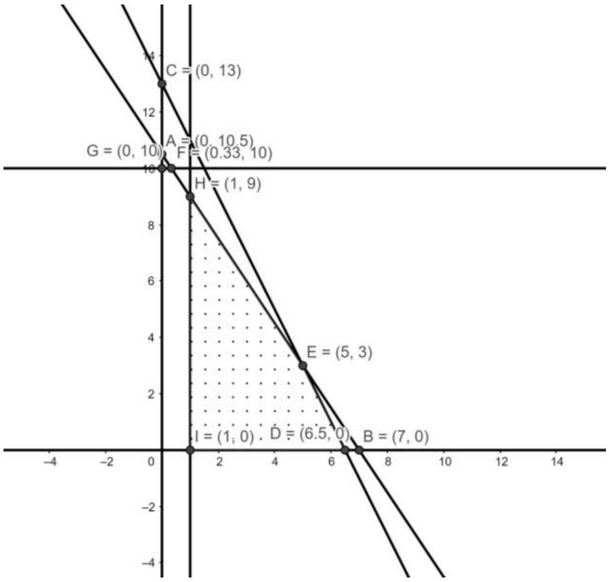

Figure 5. Program-5

Figure 6 shows the entire branching process from Program-1 to Program-5. The solution of decision variables in the form of integers is found in Program-4 and Program-5. However, when comparing the value of the two functions, the value of the Program-5 function is greater than Program-4. As a result, Program-5 is the optimal solution with the value of the decision variable in the form of an integer, namely $x_1 = 1$ and $x_2 = 9$ with a maximum profit of $z = 31$ (in ten thousand). Thus, the optimal amount of daily production at Ayyumnah store is 1 pcs of long shirt and 9 pcs of tunics with maximum profit of Rp 310.000,00.

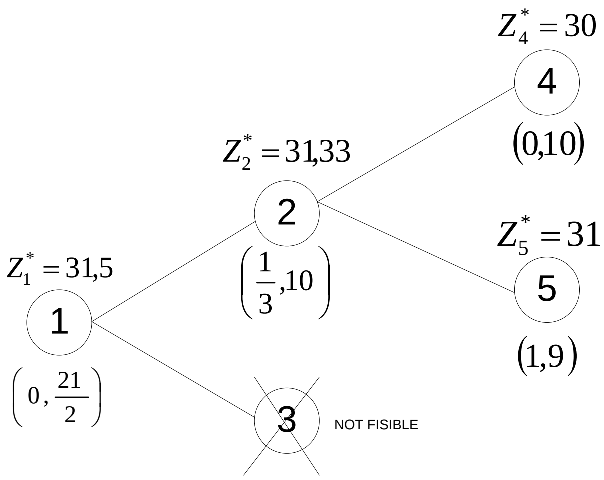

Figure 6. The entire branching process

The results of this study are in line with previous studies, namely (Buyung & Suhendar, 2020), (Buyung & Suhendar, 2021), (Jannah A.M et al., 2018), (Purba & Ahyaningsih, 2020) and (Liu & Gao, 2015). The novelty value of this study is the development of an integer linear program application using the branch and bound method in the convection sector, where this has not been discussed in previous studies. Furthermore, this research does not rule out the possibility to be developed in other sectors, for example, the broader economic sector.

CONCLUSION

Form the results and discussions, it was gained that the application of integer linear programming using the branch and bound method was in optimizing the production of Ayyummah's long shirt and tunics. Ayyumnah store must produce 1 pcs long shirt as well as 9 pcs tunics every day to achieve the maximum profit of Rp 310.000,00. The results show the application of integer programming using the branch boundary method in the home-scale convection sector.

Yet, it is possible that the research can be enhanced into the convection sector on a larger scale. Future research can develop integer linear program applications using the branch and bound method in other sectors. In addition, this research can be created using other integer linear program methods such as the cutting plane method.

AUTHOR CONTRIBUTION

Fery Firmansah conducted data processing, data analysis, manuscript writing, and manuscript submission. Muhammad Ridlo Yuwono assisted in data processing, data analysis, and language translation. Fika Aisyah Munif assisted in data collection, data processing, and graph making. All authors were responsible for completing the manuscript.

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