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Odd Harmonious Labeling of the Zinnia Flower Graphs

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ABSTRACT

An odd harmonious graph is a graph that satisfies the properties of odd harmonious labeling. In this study a new graph class construction is given, namely zinnia flower graphs and variations of the zinnia flower graphs. The research method used is qualitative and includes several stages, namely data collection, data processing and analysis, and verification of the results. The purpose of this research is to prove that the zinnia flower graph and its variations satisfy odd harmonious labeling properties. The result of this research is that the zinnia flower graph and its variations are odd harmonious graphs.

Keywords: flower graph, odd harmonious graph, odd harmonious labeling, zinnia flower graph

INTRODUCTION

The topic of research on graph labeling has grown tremendously in recent years, as evidenced by the various types of research results on graph labeling (Gallian, 2022). One of the research topics on graph labeling is odd harmonious graph labeling. Liang and Bai introduced odd harmonious graphs in 2009. Graph $G(p,q)$ with order $p=|V(G)|$ and size $q=|E(G)|$ is an odd harmonious graph if it satisfies an injective vertex labeling function $g: V(G)\rightarrow\{0,1,2,3,4,\ldots,2q-1\}$ such that it induces a bijective edge labeling function $g^*: E(G)\rightarrow\{1,3,5,7,9,\ldots,2q-1\}$ with $g^*(mn)=g(m)+g(n)$ (Liang & Bai, 2009). In the same paper, Liang and Bai proved that cycle graphs, complete graphs, bipartite graphs, and windmill graphs are odd harmonious graphs.

In a different paper, Abdel All and Seoud (2016) also found a class of odd harmonious graphs (Abdel-Aal & Seoud, 2016). Jeyanti et al in 2015 also found several classes of odd harmonious graphs (Jeyanthi et al., 2015). Other relevant research results are as follows (Abdel-Aal, 2013), (Firmansah & Yuwono, 2017a), (Firmansah, 2017), (Firmansah & Yuwono, 2017b), (Seoud & Hafez, 2018), (Jeyanthi, Philo, & Siddiqui, 2019), (Sugeng et al., 2019), (Jeyanthi, Philo, & Youssef, 2019), and (Jeyanthi & Philo, 2019).

In 2020 Febriana and Sugeng proved that odd harmonious labeling on squid graphs (Febriana & Sugeng, 2020). Sarasvati et al proved that edge combination product are odd harmonious graphs. Firmansah proved that multiple net snake graphs are odd harmonious graphs (Firmansah, 2020b). In a different paper, results of other relevant research in 2020, 2021 and 2022 are as follows (Firmansah, 2020a), (Firmansah & Tasari, 2020), (Firmansah & Giyarti, 2021), (Philo & Jeyanthi, 2021), and (Firmansah, 2022).

In this paper, we will construct the definition of the zinnia flower graph $Z(h)$ with $h \geq 1$ and variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$. Furthermore, it will be proved that zinnia flower graph $Z(h)$ with $h \geq 1$ and variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$ satisfy the properties of odd harmonic labeling. As a result, it is obtained that the zinnia flower graph $Z(h)$ with $h \geq 1$ and variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$ are odd harmonious graphs.

RESEARCH METHOD

The research in this paper is of the qualitative research type. The research stages consist of data collection, data processing and analysis, and verification of results. The data collection stage consists of collecting the latest research results on new graph construction, odd harmonious labeling, and odd harmonious graphs. The data processing and analysis stage consists of constructing definitions and properties of new graphs. The result verification stage is in the form of making theorems about odd harmonious graphs with mathematical proof.

RESULTS AND DISCUSSION

This research results in constructing the definition of the zinnia flower graphs and their variations in Definition 1 and Definition 2.

Definition 1. Zinnia flower graph $Z(h)$ with $h \geq 1$ is a graph with vertex set $V(Z(h)) = \{a_j \mid 1 \leq j \leq 2h+2\} \cup \{b_i \mid i = 1,2\} \cup \{c_j{}^i \mid 1 \leq j \leq h, i = 1,2\}$ and edge set $E(Z(h)) = \{a_j b_i \mid 1 \leq j \leq 2h+2, i = 1,2\} \cup \{a_1 c_j{}^i \mid 1 \leq j \leq h, i = 1,2\} \cup \{a_2 c_j{}^i \mid 1 \leq j \leq h, i = 1,2\}$.

Based on Definition 1, $p = |V(Z(h))| = 4h + 4$ and $q = |E(Z(h))| = 8h + 4$ are obtained and the figure construction of the zinnia flower graph $Z(h)$ is obtained as follows

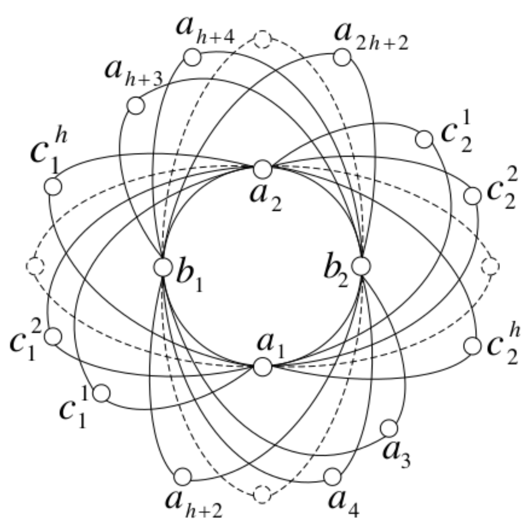

Figure 1. Zinnia flower graph $Z(h)$

Definition 2. Variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$ is a graph with vertex set $V(Z_v(h)) = \{u_0\} \cup \{v_j{}^i | i,j = 1,2\} \cup \{w_j | 1 \leq j \leq h\} \cup \{x_j | 1 \leq j \leq h\} \cup \{y_j | 1 \leq j \leq h\} \cup \{z_j | 1 \leq j \leq h\}$ and edge set $E(Z_v(h)) = \{u_0 v_j{}^i | \; i,j = 1,2\} \cup \{v_1{}^2 w_j | 1 \leq j \leq h\} \cup \{v_2{}^2 w_j | 1 \leq j \leq h\} \cup \{v_2{}^2 x_j | 1 \leq j \leq h\} \cup \{v_2{}^1 x_j | 1 \leq j \leq h\} \cup \{v_2{}^1 y_j | 1 \leq j \leq h\} \cup \{v_1{}^1 y_j | 1 \leq j \leq h\} \cup \{v_1{}^1 z_j | 1 \leq j \leq h\} \cup \{v_1{}^2 z_j | 1 \leq j \leq h\}$.

Based on Definition 2, $p = |V(Z_v(h))| = 4h + 5$ and $q = |E(Z_v(h))| = 8h + 4$ are obtained and the figure construction variations of the zinnia flower graph $Z_v(h)$ is obtained as follows

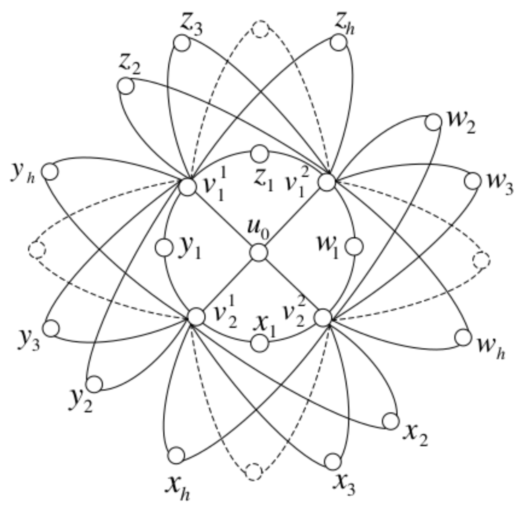

Figure 2. Variations of the zinnia flower graph $Z_v(h)$

Furthermore, it is proven that the zinnia flower graphs and their variations satisfy the properties of the odd harmonious labeling function stated in Theorem 3 and Theorem 4.

Theorem 3. Zinnia flower graph $Z(h)$ with $h \geq 1$ is an odd harmonious graph.

Proof.

Define the vertex labeling function $g: V(Z(h)) \rightarrow \{0,1,2,3 \dots, 16h + 7\}$ as follows

$$g(a_j) = 4j - 4, 1 \leq j \leq 2h + 2 \qquad (1)$$

$$g(b_i) = 2i - 1, i = 1,2 \qquad (2)$$

$$g(c_j{}^i) = 8h + 8j + 2i - 1, 1 \leq j \leq h, i = 1,2 \qquad (3)$$

Based on (1), (2) and (3), different labels are obtained and $V(Z(h)) \subseteq \{0,1,2,3 \dots, 16h + 7\}$, hence the vertex labeling function is injective.

Define the edge labeling function $g^*: E(Z(h)) \rightarrow \{1,3,5,7, \dots, 16h + 7\}$ as follows

$$g^*(a_j b_i) = 4j + 2i - 5, 1 \leq j \leq 2h + 2, i = 1,2 \qquad (4)$$

$$g^*(a_1 c_j{}^i) = 8h + 8j + 2i - 1, 1 \leq j \leq h, i = 1,2 \qquad (5)$$

$$g^*(a_2 c_j{}^i) = 8h + 8j + 2i + 3, 1 \leq j \leq h, i = 1,2 \qquad (6)$$

Based on (4), (5) and (6), different labels are obtained and $E(Z(h)) = \{1,3,5,7,\dots,16h+7\}$, hence the edge labeling function is bijective. Consequently zinnia flower graph $Z(h)$ with $h \geq 1$ is an odd harmonious graph. ∎

The zinnia flower graph $Z(4)$ as follows

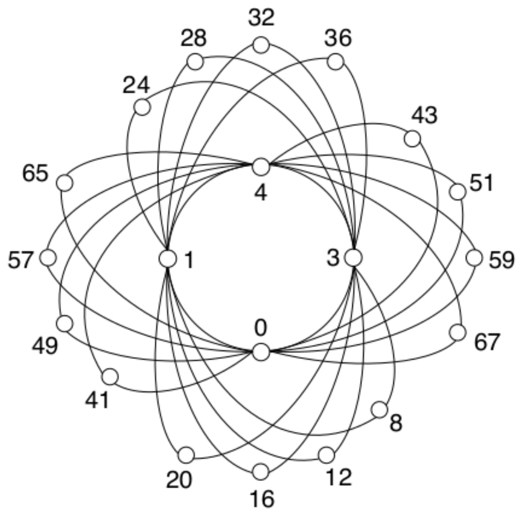

Figure 3. Zinnia flower graph $Z(4)$

Theorem 4. Variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$ is an odd harmonious graph.

Proof.

Define the vertex labeling function $g: V(Z_v(h)) \rightarrow \{0,1,2,3 \dots,16h+7\}$ as follows

$$g(u_0) = 0 \quad (7)$$

$$g(v_j{}^i) = 4j + 2i - 5,\ \ i,j = 1,2 \quad (8)$$

$$g(w_j) = 16j - 12, 1 \leq j \leq h \quad (9)$$

$$g(x_j) = 16j - 2, 1 \leq j \leq h \quad (10)$$

$$g(y_j) = 16j - 4, 1 \leq j \leq h \quad (11)$$

$$g(z_j) = 16j + 2, 1 \leq j \leq h \quad (12)$$

Based on (7), (8), (9), (10), (11) and (12), different labels are obtained and $V(Z_v(h)) \subseteq \{0,1,2,3 \dots,16h+7\}$, hence the vertex labeling function is injective.

Define the edge labeling function $g^*: E(Z_v(h)) \rightarrow \{1,3,5,7,\dots,16h+7\}$ as follows

$$g^*(u_0 v_j{}^i) = 4j + 2i - 5,\ \ i,j = 1,2 \quad (13)$$

$$g^*(v_1{}^2 w_j) = 16j - 7,\ \ 1 \leq j \leq h \quad (14)$$

$$g^*(v_2{}^2 w_j) = 16j - 5,\ \ 1 \leq j \leq h \quad (15)$$

$$g^*(v_2{}^2 x_j) = 16j + 5,\ \ 1 \leq j \leq h \quad (16)$$

$$g^*(v_2{}^1 x_j) = 16j + 1,\ \ 1 \leq j \leq h \quad (17)$$

$$g^*(v_2{}^1 y_j) = 16j - 1,\ 1 \le j \le h \quad (18)$$

$$g^*(v_1{}^1 y_j) = 16j - 3,\ 1 \le j \le h \quad (19)$$

$$g^*(v_1{}^1 z_j) = 16j + 3,\ 1 \le j \le h \quad (20)$$

$$g^*(v_1{}^1 z_j) = 16j + 7,\ 1 \le j \le h \quad (21)$$

Based on (13), (14), (15), (16), (17), (18), (19), (20) and (21), different labels are obtained and $E(Z_v(h)) = \{1,3,5,7,\ldots,16h+7\}$, hence the edge labeling function is bijective. Consequently variations of the zinnia flower graph $Z_v(h)$ with $h \ge 1$ is an odd harmonious graph. ■

Variations of the zinnia flower graph $Z(5)$ as follows

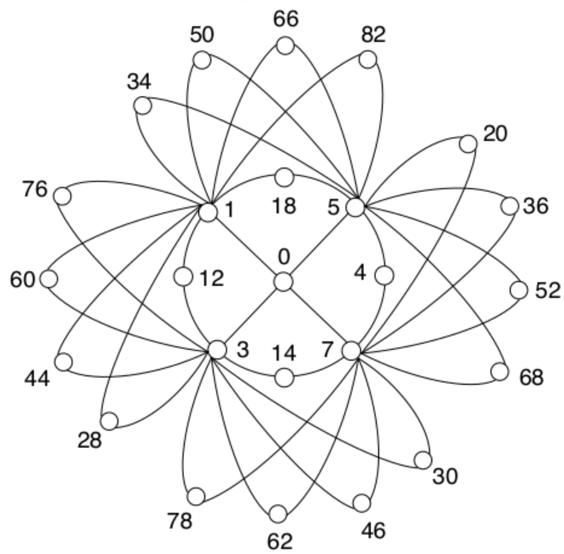

Figure 4. Variations of the zinnia flower graph $Z_v(5)$

Based on Theorem 3 and Theorem 4, it has been obtained that $Z(h)$ with $h \ge 1$ in Definition 1 and variations of the zinnia flower graph $Z_v(h)$ with $h \ge 1$ are odd harmonious labeling.

CONCLUSION

The conclusion of this research is the definition of the zinnia flower graph $Z(h)$ with $h \ge 1$ in Definition 1 and variations of the zinnia flower graph $Z_v(h)$ with $h \ge 1$ in Definition 2. In addition, the two new classes of graphs have been shown to satisfy the properties of odd harmonious graphs. Theorem 3 for the proof of the zinnia flower graph and Theorem 4 for the proof of the zinnia flower graph variation.

For further research, this study can be continued by looking for the construction of new graph class definitions that are the development of the zinnia flower graph and proving that these graphs are also odd harmonious graphs.

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