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Odd harmonious labeling on the amalgamation of the generalized double quadrilateral windmill graph

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ABSTRACT

Graph labeling is one of the topics of graph theory that is growing very rapidly both in terms of theory and application. A graph that satisfies the labeling property of odd harmonious is called an odd harmonious graph. The method used in this research is qualitative research by developing a theory and a new class of graphs from odd harmonious graphs. In this research, a new graph class construction will be given in the form of an amalgamation of the generalized double quadrilateral windmill graph. Furthermore, it will be proved that the amalgamation of the generalized double quadrilateral windmill graph is an odd harmonious graph. So that the results of the research show that the amalgamation of the generalized double quadrilateral windmill graph is a new graph class of odd harmonious graphs.

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INTRODUCTION

One of the research topics on graphs that is developing very rapidly both in terms of theory and application is graph labeling. Graph labeling is basically labeling vertex, edge, or a combination of both with certain properties.

One of the graph labeling types is odd harmonious graph labeling which was discovered by Liang & Bai (2009). A graph that satisfies the odd harmonious labeling properties is called an odd harmonious graph. Suppose graph $G(V,E)$ with $p = |V(G)|$ and $q = |E(G)|$ is called an odd harmonious graph if it satisfies the labeling function of the injective vertex $f: V(G) \to \{0,1,2,\dots,2q-1\}$ so as to induce a bijective edge labeling function of arc $f^*: E(G) \to \{1,3,5,\dots,2q-1\}$ with the definition of $f^*(ab) = f(a) + f(b)$ (Liang & Bai, 2009).

The following are some classes of graphs that have been found to be a family of odd harmonious graphs that are relevant to this research. The new graph class from the Cartesian product operation is an odd harmonious graph (Firmansah & Yuwono, 2017a), In a different paper, we obtained odd harmonious labeling on the pleated of the Dutch windmill graphs (Firmansah & Yuwono, 2017b), the odd harmonious labeling on variation of the double quadrilateral windmill graphs (Firmansah, 2017), odd harmonious labeling of grid graphs (Jeyanthi, Philo, & Youssef, 2019), odd harmonious labeling of some classes of cycle related graphs (Renuka & Balaganesan, 2018), odd harmonious labeling of super subdivision graphs (Jeyanthi, Philo, & Siddiqui, 2019).

Firmansah & Syaifuddin (2018a) have proven that the amalgamation of the Dutch windmill graph ${C_4}^r * P_2 * {C_4}^r$ is an odd harmonious graph, furthermore, in the same paper they have also proven that the k amalgamation of the Dutch windmill graph ${C_4}^r * P_2 * \ldots * {C_4}^r$ is an odd harmonious graph.

Firmansah & Syaifuddin (2018b) have constructed a double quadrilateral windmill graph amalgamation $DQ^r * P_2 * DQ^r$ obtained from the amalgamation operation of two double quadrilateral windmill graphs DQ^r with a P_2 path graph. In the same paper, they also prove that the amalgamation of a double quadrilateral windmill graph $DQ^n * P_2 * DQ^n$ is an odd harmonious graph.

The results of this research become the basis for the authors to develop a new graph class construction, namely the amalgamation of the generalized double quadrilateral windmill graph $DQ^n * P_2 * \ldots * P_2 * DQ^n$ obtained from the amalgamation operation of m graphs of double quadrilateral graphs DQ^n and the $m-1$ of the P_2 path graph. To make it easier to write an amalgamation of the generalized double quadrilateral windmill graph, it is denoted as $mDQ^n * (m-1)P_2$ with $m \geq 1, n \geq 1$.

Furthermore, the authors also proved that the amalgamation of the generalized double quadrilateral windmill graph $mDQ^n * (m-1)P_2$ satisfies the odd harmonious labeling properties so that the amalgamation of the generalized double quadrilateral windmill graph $mDQ^n * (m-1)P_2$ is an odd harmonious graph. In such a way, it is obtained that the amalgamation of the generalized double quadrilateral windmill graph $mDQ^n * (m-1)P_2$ is a new graph class of odd harmonious graphs.

METHOD

This research is qualitative research that aims to obtain new theories and properties of odd harmonious graphs. The stages of the research are:

1) Construction of a new graph class definition along with its order and size
2) Vertex labeling construction which is injective and fulfills odd harmonious labeling properties
3) Edge labeling construction which is bijective and fulfills the odd harmonious labeling properties
4) Theorem construction is accompanied by mathematical proof.

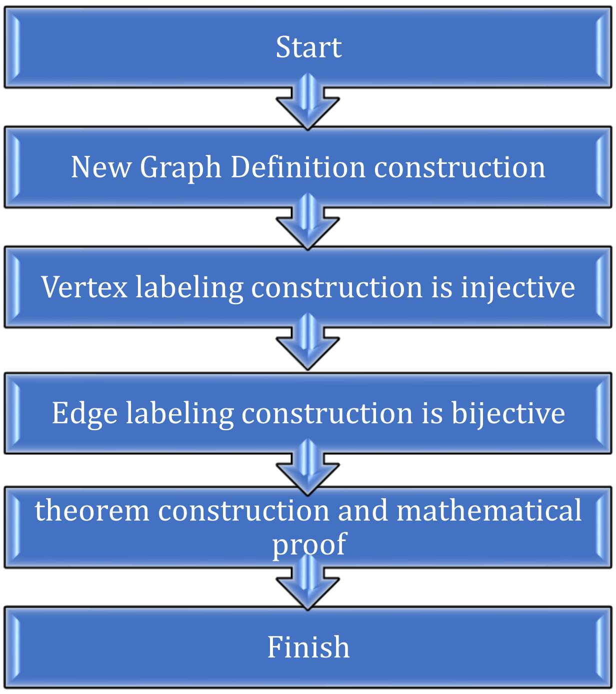

Figure 1. Research Method Flowchart

RESULTS AND DISCUSSION

Following are the results of the research in the form of the construction of an amalgamation definition of a double quadrilateral windmill graph which is generalized to Definition 1.

Definition 1. The amalgamation of the generalized double quadrilateral windmill graph $mDQ^n * (m-1)P_2$ with $m \geq 1, n \geq 1$ is the graph obtained by amalgamating m graphs of double quadrilateral graphs DQ^n and the $m-1$ of the P_2 path graph with the vertex set
$V(mDQ^n * (m-1)P_2) =$
$\{u_i | 1 \leq i \leq m\} \cup$
$\{v_i{}^j | 1 \leq i \leq m, 1 \leq j \leq 3n\} \cup$
$\{w_i{}^j | 1 \leq i \leq m, 1 \leq j \leq 2n\}$ and edge set
$E(mDQ^n * (m-1)P_2) =$
$\{u_i v_i{}^j | 1 \leq i \leq m, 1 \leq j \leq 3n\} \cup$
$\{v_i{}^{3j-2} w_i{}^{2j-1} | 1 \leq i \leq m, 1 \leq j \leq n\} \cup$
$\{v_i{}^{3j} w_i{}^{2j} | 1 \leq i \leq m, 1 \leq j \leq n\} \cup$
$\{w_i{}^{2j-1} v_i{}^{3j-1} | 1 \leq i \leq m, 1 \leq j \leq n\} \cup$
$\{w_i{}^{2j} v_i{}^{3j-1} | 1 \leq i \leq m, 1 \leq j \leq n\} \cup$
$\{u_i u_{i+1} | 1 \leq i \leq m-1\}$.

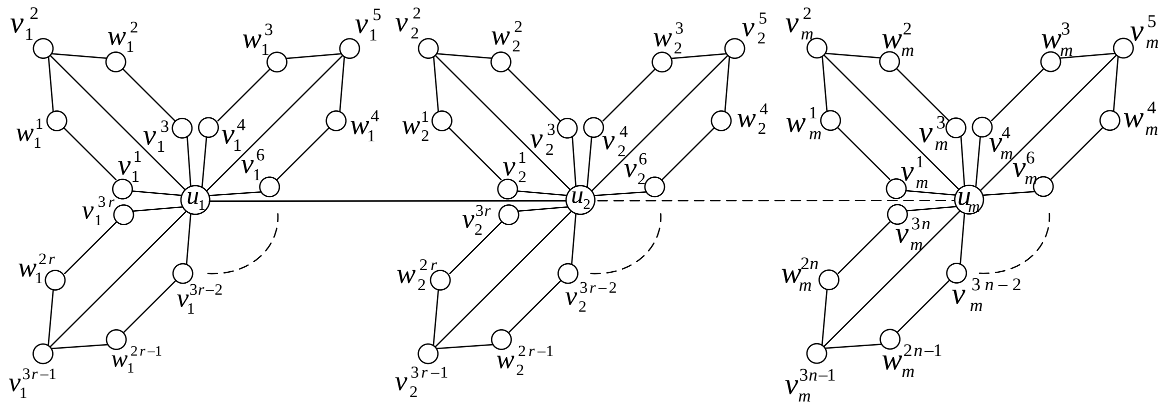

Figure 2. Graph $\boldsymbol{mDQ^n * (m-1)P_2}$

Furthermore, the amalgamation of the generalized double quadrilateral windmill graph is an odd harmonious graph as proven in Theorem 2.

Theorem 2. The amalgamation of the generalized double quadrilateral windmill graph $mDQ^n * (m-1)P_2$ with $m \geq 1, n \geq 1$ is an odd harmonious graph.

Prove.

Based on Definition 1 it is obtained that $p = |V(mDQ^n * (m-1)P_2)| = 5rk + k$ and $q = |E(mDQ^n * (m-1)P_2)| = 7rk + k - 1$.

Defined the labeling function of vertex $f: V(mDQ^n * (m-1)P_2) \to \{0,1,2,3 \ldots, 14mn + 2n - 3\}$

$$f(u_i) = \begin{cases} (3n+1)i - (3n+1), & 1 \leq i \leq m, i = \text{odd} \\ (3n+1)i - 1, & 1 \leq i \leq m, i = \text{even} \end{cases} \quad (1)$$

$$f(v_i{}^j) = \begin{cases} (3n+1)i + 2j - (3n+2), & 1 \leq i \leq m, i = \text{odd}, 1 \leq j \leq 3n \\ (3n+1)i + 2j - (6n+2), & 1 \leq i \leq m, i = \text{even}, 1 \leq j \leq 3n \end{cases} \quad (2)$$

$$f(w_i{}^j) = \begin{cases} (6n+2)k + (5n-1)i - 7j + (3n-2), & 1 \leq i \leq m, i = \text{odd}, 1 \leq j \leq 2n, j = \text{odd} \\ (6n+2)k + (5n-1)i - 7j + (3n+7), & 1 \leq i \leq m, i = \text{odd}, 1 \leq j \leq 2n, j = \text{even} \\ (6n+2)k + (5n-1)i - 7j + (6n-2), & 1 \leq i \leq m, i = \text{even}, 1 \leq j \leq 2n, j = \text{odd} \\ (6n+2)k + (5n-1)i - 7j + (6n+7), & 1 \leq i \leq m, i = \text{even}, 1 \leq j \leq 2n, j = \text{even} \end{cases} \quad (3)$$

Based on (1), (2), and (3), $f\big(V(mDQ^n * (m-1)P_2)\big) \subseteq \{0,1,2,3 \ldots, 14rk + 2k - 3\}$ is obtained and assign a different label to each edge so that the edge labeling function satisfies the injective property.

Next, define the edge labeling function $f^*: E(mDQ^n * (m-1)P_2) \to \{1,3,5,7, \ldots, 14rk + 2k - 3\}$ as follows.

$$f^*(u_i v_i{}^j) = (6r+2)i + 2j - (6r+3), 1 \le i \le k, 1 \le j \le 2r \quad (4)$$
$$f^*(v_i{}^{3j-2} w_i{}^{2j-1}) = (6r+2)k + 8ri - 8j - 1, 1 \le i \le k, 1 \le j \le r \quad (5)$$
$$f^*(v_i{}^{3j} w_i{}^{2j}) = (6r+2)k + 8ri - 8j + 5, 1 \le i \le k, 1 \le j \le r \quad (6)$$
$$f^*(w_i{}^{2j-1} v_i{}^{3j-1}) = (6r+2)k + 8ri - 8j + 1, 1 \le i \le k, 1 \le j \le r \quad (7)$$
$$f^*(w_i{}^{2j} v_i{}^{3j-1}) = (6r+2)k + 8ri - 8j + 3, 1 \le i \le k, 1 \le j \le r \quad (8)$$
$$f^*(u_i u_{i+1}) = (6r+2)i - 1, 1 \le i \le k-1 \quad (9)$$

Based on (4), (5), (6), (7), (8) and (9), $f^*\big(E(mDQ^n * (m-1)P_2)\big) = \{1,3,5,7, \ldots, 14rk + 2k - 3\}$ is obtained and assign a different label to each arc so that the arc labeling function satisfies the bijective property. As a result, the amalgamation of the generalized double quadrilateral windmill graph $mDQ^n * (m-1)P_2$ with $m \ge 1, n \ge 1$ is an odd harmonious graph. ■

To make it easier to understand, here is an example of an odd harmonious graph $6DQ^4 * 5P_2$.

Based on Definition 1 and Theorem 2, obtained an amalgamation of the generalized double quadrilateral windmill graph $mDQ^n * (m-1)P_2$ with $m \ge 1, n \ge 1$ is an odd harmonious graph, with these results showing that there is a development of graph theory. odd harmoniouss, especially the amalgamation of the double quadrilateral windmill graph which is generalized to the amalgamation of the m graph, the double quadrilateral graph DQ^n and the $m-1$ of the P_2 path graph.

CONCLUSIONS AND SUGGESTIONS

Based on the results and discussion, it is obtained that the construction of an amalgamated definition of the generalized double quadrilateral windmill graph $mDQ^n * (m-1)P_2$ with $m \ge 1, n \ge 1$. Furthermore, it is found that the amalgamation of the generalized double quadrilateral windmill graph $mDQ^n * (m-1)P_2$ with $m \ge 1, n \ge 1$ is an odd harmonious graph.

The results of this research can be continued to find a new class of graph which is a family of odd harmonious graphs. Suppose one is looking for odd harmonious labeling of $mDQ^n * (m-1)P_k$ with $m \ge 1, n \ge 1$, and $k \ge 1$.

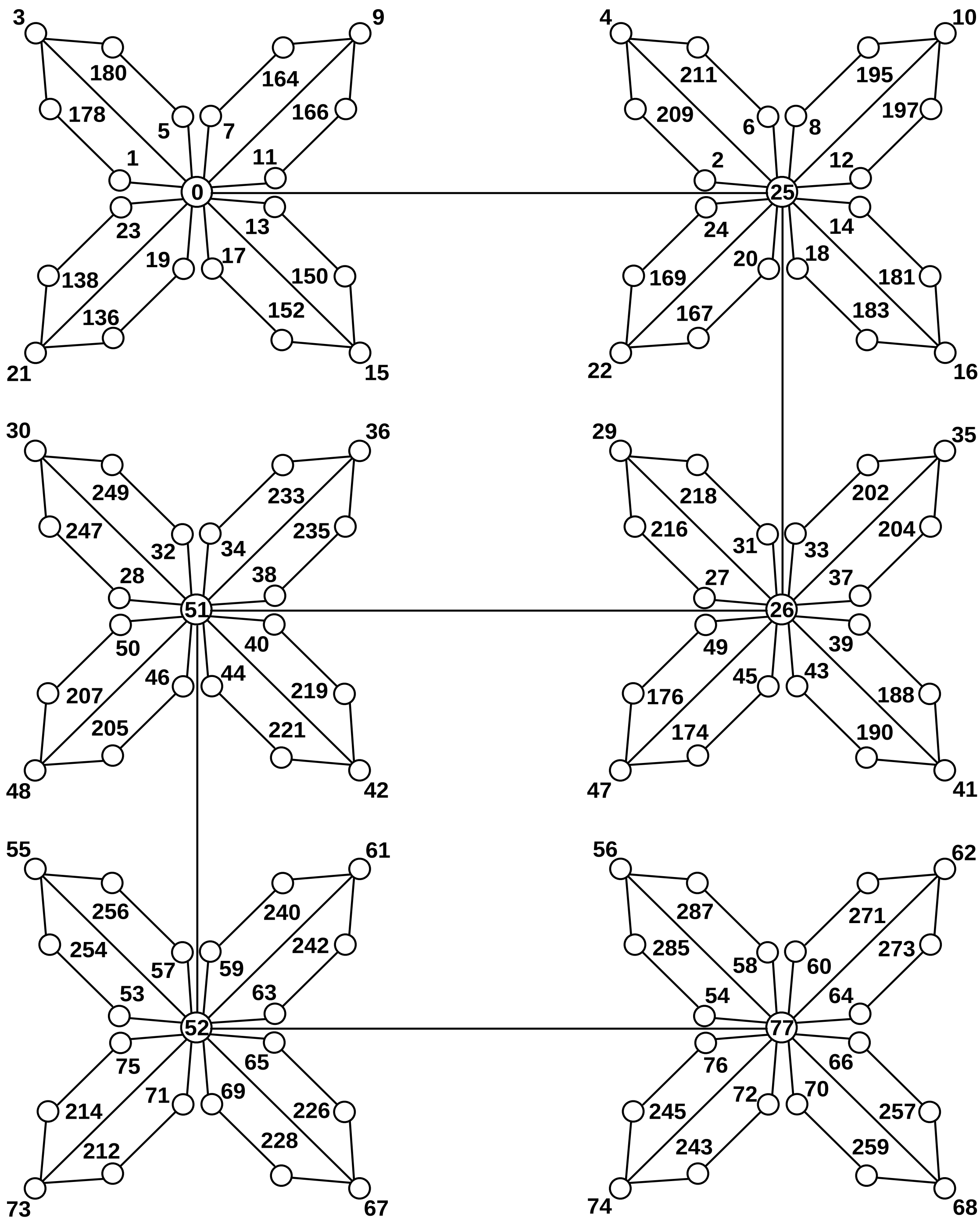

Figure 3. Odd Harmonious Graph $6DQ^4 * 5P_2$

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